# THE CALCULATION OF Solar Eclipſes WITHOUT PARALLAXES.

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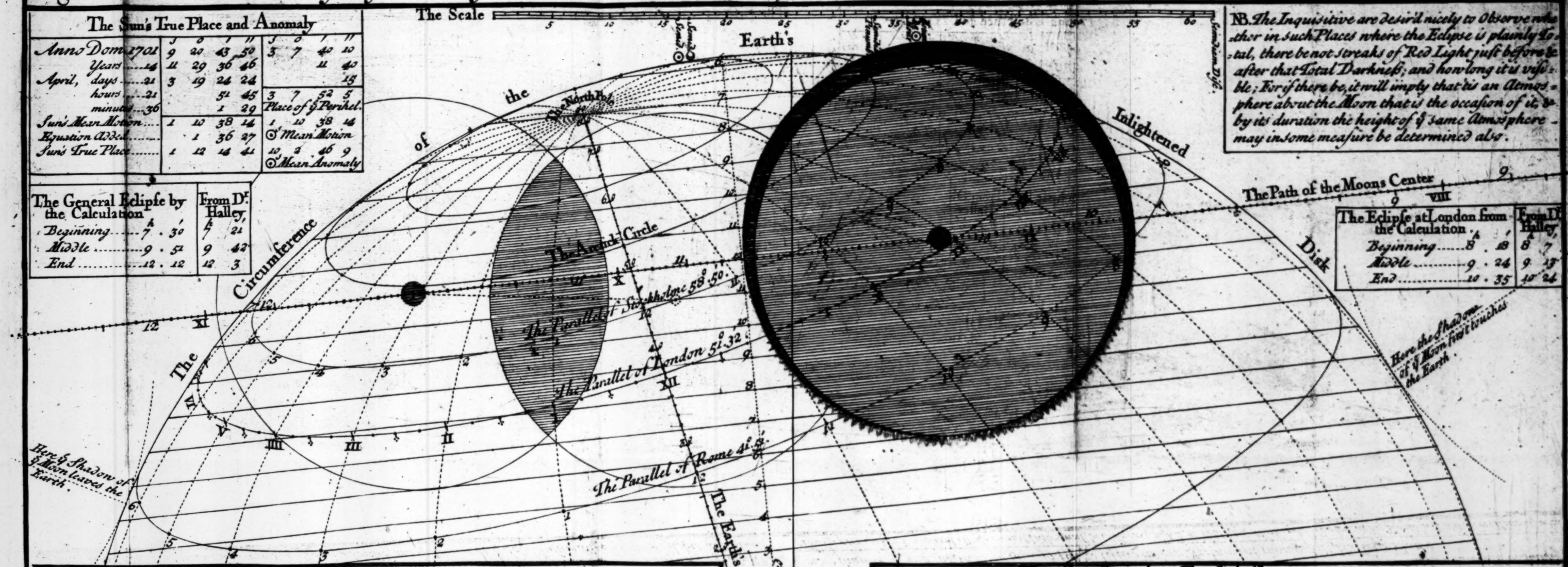
# A Calculation of the Great Eclipſe of the Sun, *April 22d. 1715 in ye Morning*, from Mr. Flamſteed's Tables; as corrected accord-ing to Sr. Iſaac Newton's Theory of ye Moon in ye Aſtronomical Lectures; with its Construction for *London Rome and Stockholme*. By W: Whiſton M.A

| The Sun's True Place and Anomaly | s | ° | ' | " | s | ° | ' | " |
|---|---|---|---|---|---|---|---|---|
| *Anno Dom.* 1701 | 9 | 20 | 43 | 50 | 3 | 7 | 40 | 10 |
| *Years* 14 | 11 | 29 | 36 | 46 | | | 11 | 40 |
| *April, days* 21 | 3 | 19 | 24 | 24 | | | | 15 |
| *hours* 21 | | | 51 | 45 | 3 | 7 | 52 | 5 |
| *minutes* 36 | | | 1 | 29 | *Place of ye Perihel.* | | | |
| *Sun's Mean Motion* | 1 | 10 | 38 | 14 | 1 | 10 | 38 | 14 |
| *Equation Added* | | 1 | 36 | 27 | *☉ Mean Motion* | | | |
| *Sun's True Place* | 1 | 12 | 14 | 41 | 10 | 2 | 46 | 9 |
| | | | | | *☉ Mean Anomaly* | | | |

| The General Eclipſe by the Calculation | h | ' | From Dr. Halley h | ' |
|---|---|---|---|---|
| *Beginning* | 7 | 30 | 7 | 21 |
| *Middle* | 9 | 51 | 9 | 42 |
| *End* | 12 | 12 | 12 | 3 |

*NB. The Inquisitive are desir'd nicely to Obſerve whether in ſuch Places where the Eclipse is plainly Total, there be not ſtreaks of Red Light juſt before & after that Total Darkneſs; and how long it is visible; For if there be, it will imply that tis an Atmosphere about the Moon that is the occasion of it, & by its duration the height of ye same Atmosphere may in some meaſure be determined also.*



| The Eclipſe at London from the Calculation | h | ' | From Dr. Halley h | ' |
|---|---|---|---|---|
| *Beginning* | 8 | 18 | 8 | 7 |
| *Middle* | 9 | 24 | 9 | 13 |
| *End* | 10 | 35 | 10 | 24 |

| | Moon's mean Motion | | | | Motion of the Apogee | | | | Motion of ye Node Retr. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | s | ° | ' | " | s | ° | ' | " | s | ° | ' | " |
| *Anno Dom.* 1701 | 10 | 15 | 19 | 50 | 11 | 8 | 18 | 20 | 4 | 27 | 24 | 20 |
| *Years* 14 | 1 | 20 | 54 | 34 | 6 | 29 | 37 | 50 | 9 | 0 | 45 | 35 |
| *April, days* 21 | | 22 | 34 | 47 | | 12 | 21 | 59 | | 5 | 52 | 41 |
| *hours* 21 | | 11 | 31 | 46 | | | 5 | 51 | | | 2 | 47 |
| *minutes* 36 | | | 19 | 46 | | | | 10 | | | | 5 |
| *(Mean Time)* | | | | | | | | | 9 | 6 | 41 | 8 |
| *Moon's mean Motion* | 1 | 10 | 40 | 43 | 6 | 20 | 24 | 10 | 7 | 20 | 43 | 12 |
| *Suns mean Anomaly* | 10 | 2 | 46 | 9 | *An: Eq: Add.* | | 16 | 8 | *An Eq: subst.* | | 7 | 41 |
| *Physical parts Substracted* | | | 9 | 50 | 6 | 20 | 40 | 18 | 7 | 20 | 35 | 31 |
| *2d. & 6th. Equation Substract.* | | | 3 | 45 | *Mean Place Corrected* | | | | *Mean Place Corrected* | | | |
| | | *Sum* | 13 | 35 | 1 | 12 | 14 | 41 | 1 | 12 | 14 | 41 |
| *Moon's mean Place corrected* | 1 | 10 | 27 | 8 | *Suns True Place* | | | | *Sun's True Place* | | | |
| *True Place of ye Apogee subst.* | 6 | 27 | 56 | 5 | 6 | 21 | 34 | 23 | 5 | 21 | 39 | 10 |
| *Moon's mean Anomaly* | 6 | 12 | 31 | 3 | *Annual Argument* | | | | *☉ Diſtance from ye Node* | | | |
| *Equation Added* | | 1 | 40 | 35 | | 7 | 15 | 47 | *Equat: subst.* | | 25 | 19 |
| *Moon's Equat Place in its Orb.* | 1 | 12 | 9 | 43 | *Equation Added* | | | | 7 | 20 | 10 | 12 |
| *Sun's True Place* | 1 | 12 | 14 | 41 | 6 | 27 | 56 | 5 | *True Place of the Node* | | | |
| *Moon's Diſtance from ye Sun* | 11 | 29 | 55 | 2 | *True Place of ye Apogee* | | | | | 5 | 16 | 56 |
| *Variation Subſtracted* | | | | 2 | *Eccentricity 6368* | | | | *Inclination of ye Limit* | | | |
| *Moon's True Place in its Orbit* | 1 | 12 | 9 | 41 | *☽ horary Mot.* | | 38 | 0 | *Semidiam. ☉* | | 15 | 58 |
| *Node's True Place Subſtract.* | 7 | 20 | 10 | 12 | *☉* | | 2 | 25 | *Semidiam. ☽* | | 16 | 47 |
| *Argument of Latitude* | 5 | 21 | 59 | 29 | *☽ from ☉* | | 35 | 35 | *Semid. Penum.* | | 32 | 45 |
| *Reduction Added* | | | 2 | 0 | *35 6 : 60 :: 5 : 8* | | | 22 | *Semid. Diſk* | | 61 | 30 |
| *Moon's True Place in ye Eclip.* | 1 | 12 | 11 | 41 | *Eq. Time Add.* | | 3 | 25 | *Ang: of ye ☽ way with ye Ecliptic* | | 5° | 35' |
| *Moon's True North Latitude* | | | 44 | 10 | *Reduct Add.* | | 3 | 9 | *Diff: of ye ☉ & ☽ Diam.* | | | |
| | | | | | *Sum of 3 Add.* | | 14 | 56 | *98 or 98 Miles.* | | | |

*So that ye Middle of the General Eclipſe in common or apparent Time, will be 50'. 56". after Nine in the Morning; differing from Dr. Halley's Computation near 9 min. But Note that the Conſtruction is accommodated to the Drs Calculation.*

*Note also that hence the breadth of ye Shadow of Total Darkneſs will be 98 Geographical Miles; and that its length on the Oblique Horizon of London will be near 150 Miles, as Dr. Halley's Deſcription aſserts.*

*But it muſt be here Obſerved that if in this Calculation ye 2d. and 6th New Equations of the Moon, taken from Sr. Isaac Newton's Theory, were neglected, this Calculation would be much nearer to Dr. Halley's, as it is now nearer to Mr. Flamſteed's. This Eclipſe, if the Air prove clear for exact Obſervations, will go a great way to determin how far those Equations are juſt; and how far they are neceſsary in the Calculation of the New and Full Moons, and of Eclipses, that happen only at those Times. Sr. Isaac Newton's third Equation, wch is no more than 13" to be substracted, is here omitted, as very inconsiderable.*

## The Conſtruction Explaind

*This Scheme represents one half of the Inlightened Disk of ye Earth as seen from its Center Projected at ye distance of the Moon. The Elliptick Parallels, with their Hours, represent the Cities of* **London**, *&* **Rome**, *and* **Stockholme**, *as plac'd at those Hours at different Times. The Principal strait Line divided by dotts represents the Path of the Moons Center over the Disk of the Earth: And by the Hours in Larger and those above and below in smaller Characters, the Position of the Center is determind at those Times for those Places reſpectively. So that if with a pair of Compaſses we take from the proper Scale the Semidiameter of the Penumbra, and carry it along the Path till it first reaches to, and then leaves the same minutes on any Parallel ye is ye very time of its Begining and Ending there. And if at any intermediate time in both you make Circles, one with the Moons Semidiameter on its Path; the other with the Suns on any of the 3 Parallels, the Intercepted part will shew the quantity of the Eclipse at that time in the Place to which the Parallel you use does belong. And if you carry a Square along the Path, till the Perpendicular side cuts the same Hour and Minute there and in any Parallel, that is the Middle of ye Eclipse there. Of all which you have examples in the Scheme. Only Note that the Center of the Penumbra at 21' after 7 and at 3' after 12, which are the beginning and ending of the General Eclipse, extends beyond the Copper Plate, and is to be supply'd by the Pen at ye interſection of ye proper Lines there to directed.*

*The Breadth of the intire Penumbra or partial Eclipse upon this Perpendicular Plain, appears by the Construction to be no leſs than 1965 minutes or Geographical Miles on each side of ye Moons Path, or 3930 Miles in all; wch correspond to many more on ye Spherical Surface of ye Earth: Nor is it all confin'd, as you may see here, to ye Surface, but reaches off a great way into ye empty Space beyond it, Northward. The Lines which diſtinguiſh that breadth on each side into 12 Parts denote so many Digits of ye Suns Eclipse, (besides 1/3 for ye Total Shade) & ye Places both as to Long: & Lat: where ye Sun will at any Time be so much Eclipsed: And indeed I would willingly have procured a general Map here to have shew'd over what Countries and Places the intire Shadow would paſs, as Doctor Halley has given us a particular Map of England for the Paſsage of the Total Shadow over it. But the nature of the Construction does not admit of that Projection; (Such a Thing cannot be truly repreſented any other way than by the Copernicus; where there is a real Globe of the Earth, capable of a Diurnal motion, during the time of the Eclipse) the impoſsibility of which in all Perſpective Projections of the Sphere renders that designe otherwise impracticable: Nor can I determin by this Construction whether the Eclipse will be Total at London or not, because the Circles of the Sun and Moon at the Southern Limit seem here exactly coincident. But if we go by a Construction according to our Calculation ye Digits Eclipsed at London will be hardly more than 11 1/3 and the Shadow will go full 30 Miles more Northward than in Dr. Halleys Map.*

*Engrav'd and Sold by John Senex at ye Globe in Salisbury Court near Fleet Street. And Will: Taylor at ye Ship in Paternoster Row. Where are sold Mr. Whiston's Astronomical Lectures, his Tacquet's Euclid, and ye Scheme of ye Solar System. Also ye newest Globes and Maps.*

*The Copernicus, or Universal Aſtronomical Instrument, being now finish'd, is sold by ye Author Mr. Whiston, at his House in Croſs Street Hatton Garden*

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# THE CALCULATION OF *Solar Eclipſes* WITHOUT PARALLAXES.

WITH

A SPECIMEN of the ſame in the Total Eclipſe of the Sun, *May* 11. 1724.

*Now firſt made Publick.*

To which is added,

A PROPOSAL how, with the Latitude given, the Geographical Longitude of all the Parts of the Earth may be ſettled by the bare Knowledge of the Duration of Solar Eclipſes, and eſpecially of Total Darkneſs.

WITH

An ACCOUNT of ſome late Obſervations made with Dipping Needles, in order to diſcover the LONGITUDE and LATITUDE at Sea.

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By *WILL. WHISTON*, M. A.
Sometime Profeſſor of the Mathematicks in the Univerſity of *Cambridge*.

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W. TAYLOR in *Pater-Noſter-Row*. 1724.

# LEMMATA:

# OR,

# Preparatory Propoſitions.

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## I.

THE moſt uſeful and moſt remarkable *Cycle* or *Period* for the Revolution of Eclipſes, both Solar and Lunar, mentioned by *Pliny*, ( Nat. Hiſt. II. ) and by him only of all the Ancients, is the Interval of 223 Synodical Months = 6585 Days: or = 18 *Julian* Years: with 10 Days, when the Cycle or Period contains 5 Leap-Years: and with 11 Days, when 4: with 7 Hours 43′¼. In which Time the direct mean Motion of the *Moon* and her *Apogee* in the Ecliptick, is nearly ſo much *more*; and the Retrograde mean Motion of the *Nodes* nearly ſo much *leſs* than entire Revolutions, as the mean Motion of the *Moon from the Sun*, upon which all mean

Conjunctions, Oppoſitions, and Eclipſes properly depend, exceeds the like entire Revolution; which Conſtitutions do therefore nearly reſtore the mean State of the Moon it ſelf, its Apogee, Nodes, and [illegible]: And produce an eminent Revolution of correſpondent New Moons, Full Moons, and Eclipſes after that Interval perpetually.

This appears by the following Calculation of all theſe mean Motions from the Aſtronomical Tables.

*Mean Motion of the Sun or Earth.*

| | | s | ° | ' | " |
|---|---|---|---|---|---|
| Years | 18 | 11 | 29 | 28 | 32 |
| Days | 11 | 0 | 10 | 50 | 32 |
| Hours | 7 | 0 | 0 | 17 | 15 |
| ' | 43 | 0 | 0 | 1 | 46 |
| " | 15 | 0 | 0 | 0 | 1 |
| Sum | | 0 | 10 | 48 | 6 |

*Mean Motion of the Moon in the Ecliptick.*

| | | s | ° | ' | " |
|---|---|---|---|---|---|
| Years | 18 | 7 | 11 | 37 | 22 |
| Days | 11 | 4 | 24 | 56 | 25 |
| Hours | 7 | 0 | 03 | 50 | 35 |
| ' | 43 | 0 | 00 | 23 | 36 |
| " | 15 | 00 | 00 | 00 | 08 |
| Sum | | 00 | 10 | 48 | 06 |

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### *Mean Motion of the Apogee.*

| | | | | | |
|---|---|---|---|---|---|
| Years | 18 | 0 | 12 | 23 | 53 |
| Days | 11 | 0 | 1 | 13 | 32 |
| Hours | 7 | 0 | 0 | 1 | 57 |
| ' | 43½ | 0 | 0 | 0 | 12 |
| Sum | | 0 | 13 | 39 | 34 |

### *Mean Motion of the Node backward.*

| | | | | | |
|---|---|---|---|---|---|
| Years | 18 | 11 | 18 | 7 | 39 |
| Days | 11 | 0 | 0 | 34 | 57 |
| Hours | 7 | 0 | 0 | 0 | 56 |
| ' | 43¼ | 0 | 0 | 0 | 6 |
| Sum | | 11 | 18 | 43 | 38 |

### *Mean Motion of the Moon from the Sun.*

| | | s | ° | ' | '' |
|---|---|---|---|---|---|
| Years | 18 | 7 | 11 | 58 | 50 |
| Days | 11 | 4 | 14 | 5 | 53 |
| Hours | 7 | 0 | 3 | 33 | 20 |
| ' | 43 | 0 | 0 | 21 | 50 |
| '' | 15 | 0 | 0 | 0 | 7 |
| Sum | | 0 | 0 | 0 | 0 |

From

| | | ° | ′ | ″ |
|---|---|---|---|---|
| From the mean Motion of the Apogee — | | 13 | 39 | 34 |
| Subſtract that of the Moon in the Eclipt. - | | 10 | 48 | 6 |
| Remains ——— (2°⌞9) ——— ——— | | 2 | 51 | 28 |
| To the mean Motion of the Moon — | 0 | 10 | 48 | 6 |
| Add that of the Node ——— | 11 | 18 | 43 | 38 |
| Sum ——— | 11 | 29 | 31 | 44 |
| Difference from 12 Signs (0°⌞47) — | 0 | 0 | 28 | 16 |

Whence it appears that the Difference of the mean Motions of the Apogee and of the Nodes from that of the Moon her ſelf in the Ecliptick, in ſuch a Period, is but ſmall: Not more in the former Caſe than 2° 51′ 28″ = 2°⌞9, nor in the latter than 28′ 16″ = 0°⌞47. Whence alſo it appears that the Lunar Apogee does in every ſuch Cycle differ but $\frac{1}{62}$ of the entire Difference 2⌞9) 180 (62. That the Lunar Node only differs in that Cycle $\frac{1}{383}$ of the entire Difference 0°⌞47) 180 (383. And that the Anomaly of the Sun it ſelf differs only $\frac{1}{16\llcorner 6}$ of the entire Anomaly 10⌞8) 180 (16⌞6. Which Quantities being generally ſmall, cannot occaſion any great Inequality in the Times and Circumſtances of New and Full Moons, or of Eclipſes; nor by conſequence greatly diſturb the regular Succeſſion of the ſame in any ſingle Period; nor indeed very greatly in ſeveral ſucceſſive Periods. For ſince the mean Motion of the *Moon from the Sun* is within a very ſmall Matter ever certain and invariable, that Revolution is always juſt; and always determines the mean Time of all Conjunctions, Oppoſitions, and Eclipſes rightly; and ſince the other Anomalies are but ſmall, and always come right again in Length of Time, they cannot ever produce any very great Anomalies in our Calculations from them. As will farther appear under the following *Scholia*.

*N. B.* This Period for Eclipſes, has of late been called, both by Mr. *Flamſteed*, and Dr. *Halley*, the *Saros*, or the *Chaldean Saros*: As if it were known and us'd by the old *Chaldeans*, and thence called by that Name. For which I know no ſufficient Foundation. There is indeed a groſs Miſtake of *Pliny*'s Number in *Suidas*, (who thus applies this term) 222 for 223 Months, as almoſt all the Editions of *Pliny* ſtill have it; and He calls that Period by this *Chaldean* Name *Saros*. Yet the *Chaldeans* never, that we find, apply'd it to any other Period than that of 3600 *Years* or *Days*; by which Period alone all the Antediluvian Reigns are determined both in *Abydenus* and *Beroſus* themſelves, from the ancienteſt Records of that Kingdom. See my *New Theory*, the later Editions, *Hypoth.* X. *Lem.* to the third Argument; and *Appendix* to the *Eſſay* towards reſtoring the true Text of the Old Teſtament, *p.* 203,—— 213.

## *SCHOLIA.*

(1.) We may here Obſerve, that ſince the Limit for Eclipſes of the Moon is about 11° 40′ = 700′ on each Side of the Node; as is the Limit for Eclipſes of the Sun, about 16° 40′ = 1000′. If we divide 700, the Limit of the Moon's Eclipſes, by $28' 16'' = 28'{\llcorner}3$, which is the Difference between the Revolution of the Moon to the Sun and of the Node above given, we ſhall have nearly Twenty Five for the Number of Cycles, after a Central Lunar Eclipſe in one of the Nodes, before the Moon goes off the Shadow of the Earth entirely at the ſame Node, and 450 Years (25 × 18 = 450,) or double that Number 900 Years for the Time that the Moon begins to enter the Ecliptick Limit on one Side, till it goes out of it on the other. During which long Interval there will ſtill be Eclipſes of the Moon each Period. And if we divide 1000′ the Limit of the Sun's Eclipſes, by the ſame Number 28′,3, we ſhall have nearly 35 for the Number of Periods after a Solar Central Eclipſe at the Middle of the Earth, in one of the Nodes, before the Penumbra goes off the *North* or *South* Parts of the Diſk of the Earth entirely at the ſame Time; *i. e.* 630 Years. (35 × 18 = 630,) or double that Number 1260 Years, from the Time that the Moon in any ſuch Period begins to enter the Ecliptick Limit on one Side, till it goes out of on the other: During which longer Interval there will ſtill be ſomewhere Eclipſes of the Sun each Period. After which reſpective long Intervals of Time there will be no ſuch Eclipſes for much longer Intervals.

(2.) Since the utmoſt Latitude of the Moon that can permit any Lunar Eclipſe, is about 62′, and the ſame utmoſt Latitude that can permit a Solar Eclipſe is about 92′: If we divide the firſt Number by 25, or the laſt by 35, the Numbers of Revolutions for the Ecliptick Limits, we ſhall have about 2′,6 = 2′: 36″ for the mean Alteration of the Moon's Latitude in each ſingle Period all along; and this both for Solar and Lunar Eclipſes. Which Latitude will be *South* during the one half of the long Period of the Ecliptick Limits before-mentioned; and *North* during the other half: Gradually increaſing, and, as gradually decreaſing perpetually: As in the following Table.

### A Table of the mean Latitudes of the Moon each ſingle Cycle, either *North* or *South*; beginning at an Eclipſe in one of the Nodes; without any Latitude at all.

***Latitudes for* 25 *Cycles in Lunar, and* 35 *in Solar Eclipſes.***

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| Cycles | ' | '' |
|---|---|---|
| 1 | 2 | 26 |
| 2 | 4 | 52 |
| 3 | 7 | 19 |
| 4 | 9 | 46 |
| 5 | 12 | 14 |
| 6 | 14 | 43 |
| 7 | 17 | 12 |
| 8 | 19 | 42 |
| 9 | 22 | 12 |
| 10 | 24 | 43 |
| 11 | 27 | 14 |
| 12 | 29 | 46 |
| 13 | 32 | 19 |
| 14 | 34 | 52 |
| 15 | 37 | 26 |
| 16 | 40 | 1 |
| 17 | 42 | 36 |
| 18 | 45 | 12 |
| 19 | 47 | 48 |
| 20 | 50 | 25 |
| 21 | 53 | 3 |
| 22 | 55 | 41 |
| 23 | 59 | 20 |
| 24 | 60 | 59 |
| 25 | 63 | 39 |
| 26 | 66 | 9 |
| 27 | 68 | 41 |
| 28 | 71 | 23 |
| 29 | 74 | 6 |
| 30 | 76 | 49 |
| 31 | 79 | 33 |
| 32 | 82 | 17 |
| 33 | 85 | 2 |
| 34 | 87 | 48 |
| 35 | 90 | 34 |

(3.) Since the principal Alteration in the Quantity and Duration of total Eclipſes of the Sun, ariſes from the Difference there is at any Time between the real Diſtances, and apparent Diameters of the Sun and Moon, at the Time of ſuch Eclipſes, that Quantity and Duration muſt depend on the Difference of their mean Anomalies, which gives us that Difference of Diſtances and Diameters; and muſt therefore anſwer in each Cycle one with another, to the Differences of thoſe mean Anomalies during that Interval; which in the Sun comes to $\frac{10^\circ,8}{180}$ or $\frac{1}{16,6}$ of its entire Anomaly. And in the Moon to $\frac{5,9}{180}$ or $\frac{1}{30}$ of its entire Anomaly. And ſince the whole mean Excentricity of the Moon is ſomewhat above three Times as great as that of the Sun, or as $\frac{55}{1000}$ to $\frac{17}{1000}$. The Differences of the Sun's Diſtances and Diameters will be but a little greater in each Period one with another; than thoſe of the Moon.

(4.) When therefore the Anomalies of the Sun and Moon are of the ſame Species; I mean both aſcending, or both deſcending; their Diſtances and Diameters will, one with another, increaſe or decreaſe nearly in the ſame Proportion; and the Quantity and Duration of total Darkneſs will alter but little in ſuch a Period. But when thoſe Anomalies are of the contrary Species; that is, the one aſcending while the other deſcends; they will alter conſiderably. So that if the Sun be deſcending, and its apparent Diameter Increaſing; while the Moon is aſcending, and its apparent Diameter Decreaſing, the Eclipſe of the Sun will, each ſucceeding Cycle, afford a ſmaller total Shadow; till at laſt it afford no total Shadow

dow at all; but the Eclipſes become *Annular.* And if the Sun be Aſcending while the Moon is Deſcending, the contrary will happen; and the total Shadow grow greater perpetually. From which Circumſtances of the Sun and Moon in each Revolution of the Cycle duly conſidered, we may nearly determine whether any ſucceeding correſpondent Eclipſe will afford us a greater or leſs total Shadow, or whether the Eclipſes will be only Annular.

(5.) From the like Circumſtances we may alſo nearly determine whether ſuch Eclipſes will come ſomewhat ſooner or later, than that of the mean Revolution of the Period before us. For if the Earth be much nearer its Aphelion, than the Moon its Apogæon, at the end of any Cycle; and by conſequence if the Earth then revolve comparatively ſlower, and the Moon ſwifter than ordinary; the meeting of the Luminaries will be accelerated. And if the Earth be much remoter from its Aphelion than the Moon from its Apogæon, the contrary will happen; and the Moon will be later than ordinary e'er it overtake the Sun. So that in the former Caſe the Eclipſe will come a little *before*, and in the latter a little *after* the proper Concluſion of that Period.

(*6.*) Since the Motion of the Node backward in one of theſe Periods does not quite reach to the Conjunction or Oppoſition, that Node muſt every Cycle go forward, with reſpect to the Lunations and Eclipſes; and at the aſcending Node the Moon will paſs more *Southward*, and at the deſcending Node more *Northward* ſucceſſively. Thus at the Solar Eclipſe *May* 1. 1706. the Moon, near its aſcending Node, had greater *Northern* Latitude than it will have at the next correſponding Solar Eclipſe, *May* 11. 1724. And thus at

the

the total Solar Eclipſe, *April* 22. 1715. the Moon near its deſcending Node, had leſs *Northern* Latitude than it will have at its correſponding great Eclipſe, *May* 2. 1733.

(7.) Since the Motion of the Moon's Apogee forward is greater in one of theſe Periods than that of the Lunations, that Apogee muſt alſo go forward every Cycle: And if at any one Solar Eclipſe that Apogee be in quadrature with the Sun, after it had been in Conjunction, the Moon will the next Period deſcend by going backward in its Eclipſes, towards the Perigee. And if at any one ſuch Apogee it be in the quadrature, after it had been in Oppoſition, it will the next Cycle aſcend: The Reverſe of all which is true in Lunar Eclipſes.

(8.) The Place and Motion of the Sun in its Ellipſis is ſo eaſily known, and that for many Ages, by bare Memory and Reflection, that a few Words will ſuffice. The Sun is now fartheſt from the Earth Eight Days after the longeſt Day; and neareſt to it Eight Days after the ſhorteſt: And its Motion about 1 Degree in 72 Years. Whence it is evident, that it has, for all the paſt Ages of Aſtronomy, been about the Summer Solſtice in our Apogee, and about the Winter Solſtice in our Perigee; if I may uſe the Terms of the *Ptolemaick* Syſtem. Nor is it therefore any Wonder that the greateſt total Eclipſes of the Sun have happened ſtill in the Summer, and the greateſt Annular ones in the Winter half Year: Since the farther the Sun is off in the Winter, the leſs muſt be its apparent Diameter, and by conſequence the greater the Exceſs of the Moon's Diameter above it. On which Exceſs alone the Greatneſs of ſuch Eclipſes depends. And the Reverſe is equally evident in the Caſe of Annular Eclipſes in Winter: Nor indeed is it very ſtrange, that Annular Eclipſes are in theſe Parts of the World ſo rarely obſerved; ſince they moſt uſually happen in Winter Days; which being ſhort, muſt afford us a proportionably ſmall Number of them. To the Inhabitants of the other Side of the Equator the Reverſe muſt happen. But thoſe Eclipſes very rarely come to our Notice.

(9.) Since this Period reaches only from the middle of one general Eclipſe to another, without regard to the Poſition of any particular Place on the Earth's Surface, ariſing from the diurnal Motion, we muſt remember that if an Eclipſe of the Sun happens at any particular Place conſiderably before Noon, it will come ſooner, and after Noon later than the proper Concluſion of this Period. Though it muſt be noted, that Eclipſes of the Moon being abſolute in their own Nature, are here wholly unconcerned; and no way ſubject to any Acceleration, Retardation, or Alteration on account of the diurnal Motion of the Earth.

(10.) The principal Alteration of the Time of the Day in all Eclipſes depends on the Exceſs of this Period above an even Number of Days; which is 7 Hours and 43′¼. So that the Cycle does naturally put every correſpondent Eclipſe later than the foregoing, almoſt 8 Hours, or one third part of a Day; which thing, by reaſon of the intervening diurnal Motion, greatly alters all Eclipſes, eſpecially Solar, not only as to the bare Time of the Day when, but alſo as to the Places on the Earth where ſuch correſpondent Eclipſe will be viſible.

(11.) If therefore we join three of theſe Cycles together, thoſe odd Hours and Minutes will amount nearly to a whole Day; and will therefore nearly bring the middle Point of the correſpondent Eclipſes to the ſame Time, in the ſame Place, and, in part, with the ſame Circumſtances as before: Which a ſingle Cycle cannot poſſibly do. Only with the Anticipation of 50′ in Time. Which three ſingle Cycles therefore of 19,756 Days, or of 54 Years; with 32 or 33 Days; I call the *Grand Cycle*. And this will be, I think, of the greateſt and readieſt uſe in remote Eclipſes of any other Period whatſoever.

Thus, for Example, there was a total Eclipſe of the Sun on *Black Monday*; as it has thence been called ever ſince, *March* 29. 1652. about Ten a Clock in the Morning. *Total*, I ſay it was in the *North* of *Ireland*, and the *Northweſt* of *Scotland*, tho' not ſo at *London*, ot the remoter Parts of *England* and *Scotland*. To this if we add one Cycle, the Time of the next correſpondent Eclipſe will thus be diſcovered:

| | y | d | h | ′ | ″ |
|---|---|---|---|---|---|
| To *A. D.* 1652. *March*, | 00 | 28 | 22 | 00 | 00 |
| Add———— | 18 | 10 | 7 | 43 | 15 |
| Sum 1670 *April* | 00 | 8 | 5 | 43 | 15 |

So that the correſpondent total Eclipſe ought to have been *A. D.* 1670. *April* the 8th 43′¼ paſt 5 a-Clock in the Evening. And becauſe the Earth was then a ſmall Matter nearer its Aphelion, than the Moon its Apogæon, the Time would be a little anticipated on that Account. But then, becauſe this Eclipſe was towards Evening, it would be much more retarded on that Account, than anticipated on the other; and the main Part of the Eclipſe would happen after Sun-ſet, and be here inviſible.

To this Time if we add another Cycle; the next correſpondent Eclipſe will in like Manner be diſcovered:

| | y | d | h | ′ | ″ |
|---|---|---|---|---|---|
| To *A. D.* 1670. *April* — | 00 | 08 | 5 | 43 | 15 |
| Add——— ——— | 18 | 11 | 7 | 43 | 15 |
| Sum 1688. *April* | 00 | 19 | 13 | 26 | 30 |

So that the next correſpondent total Eclipſe ought to have been *April* 20th, 26′½ after one in the Morning; and was therefore, to be ſure, utterly inviſible to us here. To this Time if we add another Cycle, we have the next correſpondent total Eclipſe thus:

| | y | d | h | ′ | ″ |
|---|---|---|---|---|---|
| To *A. D.* 1688 *April* | 00 | 19 | 13 | 26 | 30 |
| Add ——— | 18 | 10 | 07 | 43 | 15 |
| Sum 1706 *May* | 00 | 00 | 21 | 09 | 45 |

So that the next correſpondent total Eclipſe was to have been *A. D.* 1706. 9′¾ paſt Nine a-Clock in the Morning; which is not much before the Time when it was obſerv'd here; and was no other than that famous Eclipſe which was total at *Cadiz*, *Barcelona*, *Marſeilles*, *Geneva*, *Bern*, and *Zurich*; and became very remarkable for the raiſing of the Siege of *Barcelona* during that total Darkneſs: Though I have been inform'd by ſeveral there preſent, that it came to both Armies wholly unexpected, till the great Darkneſs of the Sky forced them to attend to it. Now this affords us alſo a remarkable Inſtance of the near Approximation of our *Grand Cycle*, both as to Time and Place. For if inſtead of tracing this correſpondent Eclipſe through the diſtinct Cycles we had at once taken our *Grand Cycle* of 54 Years, and 33 Days; we had come immediately to this Eclipſe: And by allowing the Anticipation of 50′ had been within about 20′ of the Calculation at *London*.

To this Time if we add another ſingle Cycle, we ſhall have the next correſpondent Eclipſe thus:

| | y | d | h | ′ | ″ |
|---|---|---|---|---|---|
| To *A. D.* 1706. *May*— | 00 | 00 | 21 | 09 | 45 |
| Add ———————— | 18 | 10 | 07 | 43 | 15 |
| Sum 1724 *May*——— | 00 | 11 | 04 | 53 | 00 |

So that we ought hence to expect the total Eclipſe next *May* 11, 53′ paſt 4 in the Afternoon. And becauſe both the Poſition of the Sun and Moon in their Ellipſes, and the more conſiderable Alteration from the Time of the Day, which is here much farther in the Evening than the laſt was in the Morning; oblige us to ſuppoſe about $1^{h}\frac{1}{4}$ Retardation, we hence juſtly expect that this Eclipſe will be the neareſt Total at *London* about 40′ paſt Six in the Evening; as the exacteſt Calculations do determine.

(12.) If we would now trace a few Lunar Eclipſes by this Cycle, we may do it according to the following Examples:

*A. D.* $168\frac{1}{2}$, *Feb.* 11. about 59′ paſt 10 a-Clock at Night, Mr. *Flamſteed* obſerv'd the Middle of a great and total Eclipſe of the Moon at *Greenwich*. Proceed therefore as is already directed to find the next correſpondent Eclipſe of the Moon thus:

| | y | d | h | ′ | ″ |
|---|---|---|---|---|---|
| To *A. D.* 168$\frac{1}{2}$ *February*- | 00 | 11 | 10 | 59 | 00 |
| Add ——————- | 18 | 11 | 07 | 43 | 15 |
| Sum 1$\frac{699}{700}$ *Febr*. | 00 | 22 | 18 | 42 | 15 |

So that this firſt correſpondent total Eclipſe of the Moon ought to have been *Feb.* 23. 1$\frac{699}{700}$, 42′$\frac{1}{4}$ paſt Six a-Clock in the Morning; or in the day-time, and ſo muſt needs have been in great Part to us inviſible.

| | y | d | h | ′ | ″ |
|---|---|---|---|---|---|
| To *A. D.* 1$\frac{699}{700}$, *Feb.* - | 00 | 22 | 18 | 42 | 15 |
| Add ——————- | 18 | 10 | 07 | 43 | 15 |
| Sum 17$\frac{17}{18}$ *March* — | 00 | 05 | 02 | 25 | 30 |

So that the next correſpondent total Eclipſe of the Moon ought to have been *March* 5, 17$\frac{17}{18}$. 25′$\frac{1}{2}$ after Two a-Clock in the Afternoon; which was in the day-time alſo; and ſo muſt equally with the former have been here inviſible.

| | y | d | h | ′ | ″ |
|---|---|---|---|---|---|
| To *A. D.* 17$\frac{17}{18}$, *March* — | 00 | 05 | 02 | 25 | 30 |
| Add —————— | 18 | 10 | 07 | 43 | 15 |
| Sum 1736 *March* — | 00 | 15 | 10 | 08 | 45 |

So that the next total correſpondent Eclipſe of the Moon is hence to be expected *A. D.* 1736, *March* 15. $8^{h}$ $\frac{3}{4}$ paſt 10 a-Clock at Night: which is about an Hour and half ſooner than the Calculation. Which difference we ſhall preſently find to be near the greateſt Difference that can happen.

This may alſo be equally obtain'd by one entire *Grand Cycle* of 54 Years, and 33 Days; with the fore-mentioned Anticipation of 50′ which from *Feb.* 11. $168\frac{1}{2}$ $10^h$ 59′, brings us directly to *March* 15, 1736, 9′ paſt 10 a-Clock at Night; or to ſomewhat above an Hour and half before the Calculation.

*N. B.* As to the proper Quantity of the ſeveral Alterations ariſing in each Period, which ought to be allowed for, they are nearly theſe: The Moon and Sun being about 31′ $\frac{2}{3}$ in Diameter, and the Digits of their Obſervation being 12. while the Difference of the Moon's Latitude, as we have ſeen, is about 2′ 36″ or the Twelfth Part of thoſe Diameters, it is plain that the mean natural Alteration of every Period in the ſame Circumſtances is about one Digit; though leſs in the leſſer, and greater in greater Latitudes: which in the Moon, whoſe Eclipſe is to all Spectators the ſame, holds conſtantly: And though the diurnal Motion of the Earth removes all particular Places, ſo much each Period as to render this Rule leſs obſervable in Solar Eclipſes, yet after each grand Period, which nearly reſtores their former Poſition, it will hold in a good Degree there alſo, I mean ſo as to alter about 3 Digits therein: But beſides that of the Digits eclipſed, we ought alſo to ſee what Alteration in Time may happen to each Period. Now as to the Inequality of the Sun's Motion, it is as we have ſeen $10^{\circ}{\llcorner}8$, and its greateſt Velocity is at the Earth's Perihelion, and its leaſt at its Aphelion: Its greateſt Alteration therefore muſt be in *Aphelio* and *Perihelio*, and is the Difference of the Equation belonging every where to the Addition of $10^{\circ}{\llcorner}8$, and is here 48″, which Space the Earth goes in about 20′ of Time. So that the Difference of Time on this Account, muſt each Period be ſome Quantity leſs than 20′. And as to the Inequality of the Moon'sMotion, it is alſo greateſt at the Perigee and Apogee, and its greateſt Alteration at the extreme Eccentricities of its Orbit is the Difference of the Equations at $2^\circ\ 51'\frac{1}{2}$; in Perigee and Apogee, according to thoſe extreme Eccentricities: = 9′ which the Moon uſually goes in ſomewhat leſs than 20′. So that the Difference of Time, on this Account, muſt each Cycle be ſome Quantity leſs than 20 Minutes alſo.

And now we come to the principal Alteration in Time that can happen in Eclipſes; though it belongs only to thoſe of the Sun: And that is the Time of the Day when they happen in any particular Place. Now becauſe the Center of the Moon uſually goes over an entire Diameter of the Diſk of the Earth, in about three Hours and an half, Part of which is almoſt always before, and Part after Noon: while the odd Hours of a ſmall Cycle $7^h\ 43'\frac{1}{4}$, may reach equally from a Forenoon to an Afternoon's ſucceſſive Eclipſe; 'tis poſſible, ſuch an Eclipſe may appear an Hour and three Quarters later than the Period it ſelf would determine it. Tho' uſually this Alteration will not be near ſo great; eſpecially when the Latitude of the Moon is very conſiderable. But then it is ſo eaſy to allow very nearly for this Inequality, upon a little Conſideration, that it ought not to be objected againſt the Accuracy of this Period. If for Six Hours from Noon we allow about an Hour and half; and for two Hours, three Quarters of an Hour; we ſhall not err very much from the true Time.

*Corollary*. If we would know what is the greateſt Inequality in Digits and Time in a grand Period, made up of Three common ones, or of 54 Years, 32 or 33 Days, beſides the conſtant Anticipation of 50′, we muſt ſay it may poſſibly, though it will very rarely, be almoſt thrice the Quantities already ſtated for a ſingle Cycle: excepting the laſt and principal Difference, peculiar to Solar Eclipſes: which is never much greater than that already mentioned.

*N. B.* If any are not contented to know theſe Matters by ſuch Approximations, but deſire the utmoſt Accuracy; they muſt either make uſe of Dr. *Halley*'s Equations, fitted to this Cycle, when publiſhed; or rather make uſe of Mr. *Flamſteed*'s or Dr. *Halley*'s moſt accurate Aſtronomical Tables, when publiſhed; with that Trigonometrical Calculation afterward, which I publiſh and exemplify in this Paper. In the former Part of which Work, this Cycle, with its proper Equations, will, at leaſt, ſave us the one half of our Calculation, if it will not bring us it ſelf to that utmoſt Accuracy: which indeed is hardly to be expected from it.

So that, upon the whole, If we duly conſider the particular Circumſtances of the Sun and Moon, with thoſe of the Aphelion, Apogee and Node, and with the Times of the Day or Night when the Cycle ends, and rightly apply them to this ſingle Cycle and to this grand Cycle, we ſhall be able nearly to determine the correſpondent Eclipſes with very ſmall Trouble or Calculation.

PATENT OFFICE

## II.

The Plane in which the Center of the Moon moves in Eclipſes, is not that of the Eclip- tick, but of the Orbit of the Moon, conſider'd with the Annual Motion: Or it is a Plane inclined to the Plane of the Ecliptick in an Angle of about 5° 36′. Which in the Calculation of Eclipſes is uſually ſtiled, *The Angle of the Moon's viſible Way*. This principal Plane I call, *The Lunar Plane*.

## III.

This *Lunar Plane* cuts the Sphere of the Earth, conſidered without its diurnal Motion, in a Circle whoſe Pole or Vertex is diſtant from the Pole of the Ecliptick in the ſame Angle. This Circle I call *The Lunar Circle*.

## IV.

In Eclipſes which happen at the Solſtices, and in the Nodes of the Moon's Orbit, the Diſtance of theſe Poles is exactly *Eaſtward* or *Weſtward*. In thoſe which happen at the Equinoxes and Nodes, the Diſtance is exactly *North* and *South*. But in all other Caſes it is Oblique.

## V.

The Angle of that Obliquity is always compos'd of the Diſtance of the Sun and Moon from the Solſtitial Colure; with the Diſtance of the ſame from the Nodes: And is ſometimes the *Sum*, and ſometimes the *Difference* of thoſe Quantities.

## VI.

The Diſtance between this Vertex and the Pole of the Earth, when Eclipſes happen at the Solſtices, is the *Sum* or *Difference* of the two forementioned Angles of Inclination; the one, of the Pole of the Equator and of the Pole of the Ecliptick = 23° 29′; the other of the Pole of the Ecliptick, and of the Vertex of the Lunar Circle = 5° 36′ nearly. But in all other Caſes a Spherical Triangle muſt be ſolv'd, in order to find that Diſtance: Of which hereafter.

## VII.

Since the Solſtitial Colure is a great Circle, that is alſo a Meridian, or paſſes through the Poles of the Earth and Ecliptick: And ſince beſides the Diſtance between the Vertex of the Lunar Circle and the Pole of the Earth, we ſhall want the Angle included between the Colure and that Line; this alſo muſt be obtained by the like Solution of a Spherical Triangle: Of which hereafter.

## VIII.

Since the great Circle that paſſes thro' the Poles of the Earth, and of the Lunar Circle, and that alone cuts both thoſe Circles, and their Parallels at Right Angles, That Meridian, and that alone wherein that Diſtance lies, is perpendicular to the *Path of the Moon's Center* along the other; and will determine the Point in that Path wherein the Center of the Shadow cuts that Meridian at Right-Angles and approaches neareſt of all to the Pole of the Earth: And indeed lays the Foundation of our future Calculations. This Meridian I call *The Primary Meridian.*

## IX.

The angular Diſtance about the Pole of the Earth, of the Meridian that is directed to the Sun at the middle Point of the whole Eclipſe from the primary Meridian, is compoſed of the *Sum* or *Difference* of the Angle made by the Solſtitial Colure and the primary Meridian; and of the Complement of the Sun's Right Aſcenſion at the middle of the general Solar Eclipſe. Which Angle is of the greateſt Conſequence in our future Calculations. This Angle I call *the Primary Angle.*

## X.

Since the Motion of the Center of the Shadow of the Moon, in Solar Eclipſes, is nearly even, and nearly recti-linear; ſince it is alſo in the Plane of the Lunar Circle; and is all one as if it were along a Line that touched that Circle at the Middle of the general Eclipſe, the Point of Contact; we muſt divide each Quadrant of 90 Degrees into 90 or 180 unequal Parts; but ſo that the *Difference* of the Sines of thoſe unequal Angles may be equal, and $\frac{1}{90}$ or $\frac{1}{180}$ of the entire Radius: That ſo the firſt Sine $\frac{1}{90}$ may be $\frac{1111}{100000}$; the second $\frac{2222}{100000}$; the third $\frac{3333}{100000}$, &c. and this from the Table of natural Sines, with their correſponding Arcs or Angles at the Vertex, as follows:

| Arcs. | | | | Arcs. | | | |
|---|---|---|---|---|---|---|---|
| Diff. | ° | ′ | Diff. Sines Equal. | Diff. | ° | ′ | Diff. Sines Equal. |
| 19ʟ1 | 0 | 18 | ½ | 19ʟ4 | 7 | 1 | 11 |
| 19ʟ1 | 0 | 38 | 1 | 19ʟ4 | 7 | 20 | ½ |
| 19ʟ1 | 0 | 57 | ½ | 19ʟ5 | 7 | 40 | 12 |
| 19ʟ1 | 1 | 16 | 2 | 19ʟ5 | 7 | 59 | ½ |
| 19ʟ1 | 1 | 35 | ½ | 19ʟ5 | 8 | 18 | 13 |
| 19ʟ1 | 1 | 54 | 3 | 19ʟ5 | 8 | 37 | ½ |
| 19ʟ1 | 2 | 13 | ½ | 19ʟ6 | 8 | 57 | 14 |
| 19ʟ2 | 2 | 33 | 4 | 19ʟ6 | 9 | 16 | ½ |
| 19ʟ2 | 2 | 52 | ½ | 19ʟ6 | 9 | 36 | 15 |
| 19ʟ2 | 3 | 11 | 5 | 19ʟ6 | 9 | 55 | ½ |
| 19ʟ2 | 3 | 30 | ½ | 19ʟ7 | 10 | 14 | 16 |
| 19ʟ2 | 3 | 49 | 6 | 19ʟ7 | 10 | 33 | ½ |
| 19ʟ2 | 4 | 8 | ½ | 19ʟ7 | 10 | 53 | 17 |
| 19ʟ2 | 4 | 28 | 7 | 19ʟ7 | 11 | 12 | ½ |
| 19ʟ3 | 4 | 47 | ½ | 19ʟ7 | 11 | 32 | 18 |
| 19ʟ3 | 5 | 6 | 8 | 19ʟ7 | 11 | 51 | ½ |
| 19ʟ3 | 5 | 25 | ½ | 19ʟ7 | 12 | 11 | 19 |
| 19ʟ3 | 5 | 44 | 9 | 19ʟ8 | 12 | 30 | ½ |
| 19ʟ3 | 6 | 3 | ½ | 19ʟ8 | 12 | 50 | 20 |
| 19ʟ4 | 6 | 23 | 10 | 19ʟ8 | 13 | 10 | ½ |
| 19ʟ4 | 6 | 42 | ½ | 19ʟ8 | 13 | 30 | 21 |

| Arcs. Diff. | ° | ′ | Diff. Sines Equal. |
|---|---|---|---|
| 19,8 | 13 | 49 | ½ |
| 19,9 | 14 | 9 | 22 |
| 20 | 14 | 28 | ½ |
| 20 | 14 | 48 | 23 |
| 20 | 15 | 8 | ½ |
| 20 | 15 | 28 | 24 |
| 20 | 15 | 48 | ½ |
| 20,1 | 16 | 7 | 25 |
| 20,1 | 16 | 27 | ½ |
| 20,1 | 16 | 47 | 26 |
| 20,2 | 17 | 7 | ½ |
| 20,2 | 17 | 27 | 27 |
| 20,2 | 17 | 47 | ½ |
| 20,2 | 18 | 8 | 28 |
| 20,3 | 18 | 28 | ½ |
| 20,3 | 18 | 48 | 29 |
| 20,3 | 19 | 8 | ½ |
| 20,3 | 19 | 28 | 30 |
| 20,4 | 19 | 48 | ½ |
| 20,4 | 20 | 9 | 31 |
| 20,4 | 20 | 29 | ½ |

| Arcs. Diff. | ° | ′ | Diff. Sines Equal. |
|---|---|---|---|
| 20,5 | 20 | 50 | 32 |
| 20,5 | 21 | 10 | ½ |
| 20,5 | 21 | 30 | 33 |
| 20,6 | 21 | 50 | ½ |
| 20,6 | 22 | 11 | 34 |
| 20,7 | 22 | 32 | ½ |
| 20,7 | 22 | 53 | 35 |
| 20,8 | 23 | 13 | ½ |
| 20,9 | 23 | 34 | 36 |
| 21 | 23 | 55 | ½ |
| 21 | 24 | 17 | 37 |
| 21,1 | 24 | 38 | ½ |
| 21,1 | 24 | 59 | 38 |
| 21,2 | 25 | 20 | ½ |
| 21,2 | 25 | 41 | 39 |
| 21,2 | 26 | 2 | ½ |
| 21,3 | 26 | 23 | 40 |
| 21,3 | 26 | 44 | ½ |
| 21,4 | 27 | 6 | 41 |
| 21,4 | 27 | 27 | ½ |
| 21,6 | 27 | 49 | 42 |

| | Arcs. | | Diff.Sines Equal. |
|---|---|---|---|
| Diff. | ° | ' | |
| 21,7 | 28 | 11 | ½ |
| 21,8 | 28 | 32 | 43 |
| 21,9 | 28 | 54 | ½ |
| 22,0 | 29 | 16 | 44 |
| 22,1 | 29 | 38 | ½ |
| 22,2 | 30 | 0 | 45 |
| 22,3 | 30 | 22 | ½ |
| 22,4 | 30 | 44 | 46 |
| 22,6 | 31 | 6 | ½ |
| 22,8 | 31 | 51 | 47 |
| 23,1 | 31 | 29 | ½ |
| 23,2 | 32 | 14 | 48 |
| 23,4 | 32 | 36 | ½ |
| 23,5 | 32 | 59 | 49 |
| 23,6 | 33 | 22 | ½ |
| 23,6 | 33 | 45 | 50 |
| 23,7 | 34 | 9 | ½ |
| 23,8 | 34 | 31 | 51 |
| 23,9 | 34 | 54 | ½ |
| 23,9 | 35 | 18 | 52 |
| 24,1 | 35 | 41 | ½ |

| | Arcs. | | Diff.Sines Equal. |
|---|---|---|---|
| Diff. | ° | ' | |
| 24,2 | 36 | 4 | 53 |
| 24,3 | 36 | 27 | ½ |
| 24,4 | 36 | 52 | 54 |
| 24,6 | 37 | 16 | ½ |
| 24,7 | 37 | 40 | 55 |
| 24,8 | 38 | 4 | ½ |
| 24,9 | 38 | 29 | 56 |
| 25,1 | 38 | 54 | ½ |
| 25,2 | 39 | 18 | 57 |
| 25,4 | 39 | 43 | ½ |
| 25,5 | 40 | 7 | 58 |
| 25,7 | 40 | 32 | ½ |
| 25,8 | 40 | 57 | 59 |
| 25,9 | 41 | 22 | ½ |
| 26,1 | 41 | 48 | 60 |
| 26,3 | 42 | 14 | ½ |
| 26,4 | 42 | 40 | 61 |
| 26,5 | 43 | 6 | ½ |
| 26,7 | 43 | 32 | 62 |
| 26,9 | 43 | 58 | ½ |
| 27,3 | 44 | 25 | 63 |

| Diff. | Arcs. ° | ′ | Diff. Sines Equal. |
|---|---|---|---|
| 27,6 | | | |
| | 44 | 52 | ½ |
| 27,8 | | | |
| | 45 | 19 | 64 |
| 28,2 | | | |
| | 45 | 46 | ½ |
| 28,4 | | | |
| | 46 | 14 | 65 |
| 28,5 | | | |
| | 46 | 42 | ½ |
| 28,6 | | | |
| | 47 | 10 | 66 |
| 28,7 | | | |
| | 47 | 38 | ½ |
| 28,8 | | | |
| | 48 | 6 | 67 |
| 28,9 | | | |
| | 48 | 36 | ½ |
| 29 | | | |
| | 49 | 4 | 68 |
| 30 | | | |
| | 49 | 33 | ½ |
| 30 | | | |
| | 50 | 3 | 69 |
| 30,2 | | | |
| | 50 | 33 | ½ |
| 30,4 | | | |
| | 51 | 3 | 70 |
| 30,6 | | | |
| | 51 | 33 | ½ |
| 31 | | | |
| | 52 | 4 | 71 |
| 31,5 | | | |
| | 52 | 35 | ½ |
| 32 | | | |
| | 53 | 7 | 72 |
| 32,5 | | | |
| | 53 | 39 | ½ |
| 33 | | | |
| | 54 | 12 | 73 |
| 33,5 | | | |
| | 54 | 45 | ½ |

| Diff. | Arcs. ° | ′ | Diff. Sines Equal. |
|---|---|---|---|
| 34 | | | |
| | 55 | 19 | 74 |
| 34,4 | | | |
| | 55 | 52 | ½ |
| 34,8 | | | |
| | 56 | 26 | 75 |
| 35,0 | | | |
| | 57 | 1 | ½ |
| 35,4 | | | |
| | 57 | 37 | 76 |
| 35,8 | | | |
| | 58 | 13 | ½ |
| 36,3 | | | |
| | 58 | 49 | 77 |
| 37 | | | |
| | 59 | 16 | ½ |
| 37,8 | | | |
| | 60 | 4 | 78 |
| 38,6 | | | |
| | 60 | 42 | ½ |
| 39,8 | | | |
| | 61 | 22 | 79 |
| 40,9 | | | |
| | 62 | 2 | ½ |
| 41,8 | | | |
| | 62 | 44 | 80 |
| 42,5 | | | |
| | 63 | 26 | ½ |
| 43 | | | |
| | 64 | 9 | 81 |
| 44,5 | | | |
| | 64 | 54 | ½ |
| 46 | | | |
| | 65 | 40 | 82 |
| 48 | | | |
| | 66 | 26 | ½ |
| 50 | | | |
| | 67 | 15 | 83 |
| 52 | | | |
| | 68 | 5 | ½ |
| 53 | | | |
| | 68 | 58 | 84 |

Arcs.

Arcs.

| Arcs. Diff. | ° | ′ | Diff. Sines Equal. | Arcs. Diff. | ° | ′ | Diff. Sines Equal. |
|---|---|---|---|---|---|---|---|
| 54 | | | | (86) | | | |
| | 69 | 52 | ½ | | 77 | 54 | 88 |
| 56 | | | | (97) | | | |
| | 70 | 49 | 85 | | 79 | 31 | ½ |
| 59 | | | | (116) | | | |
| | 71 | 48 | ½ | | 81 | 27 | 89 |
| 63 | | | | (69) | | | |
| | 72 | 51 | 86 | | 82 | 36 | ¼ |
| 67 | | | | (81) | | | |
| | 73 | 58 | ½ | | 83 | 57 | ½ |
| 72 | | | | (105) | | | |
| | 75 | 10 | 87 | | 85 | 42 | ¾ |
| 78 | | | | (258) | | | |
| | 76 | 28 | ½ | | 90 | 00 | 90 |

Where the Angle made by the Lunar Circle, and the Parallel of the Latitude is conſiderable, inſtead of the firſt Number 19′,1 you muſt take for $\frac{1}{10}$ the Numbers following; being in a reciprocal Proportion to the Secants of thoſe Angles.

| Angl. | |
|---|---|
| 0 | 19,1 |
| 5 | 19,1 |
| 10 | 18,8 |
| 15 | 18,4 |
| 20 | 17,9 |
| 25 | 17,3 |
| 30 | 16,5 |

D. N.

*N. B.* Where the Parallel is different from that at the very Middle of the Eclipſe ; as it uſually is ; you muſt increaſe or decreaſe the ſame Numbers 19˪1, *&c.* in the Proportion of the Coſines of the Latitude thus:

| ° | |
|---|---|
| 00 | 000 |
| 10 | 174 |
| 20 | 342 |
| 30 | 500 |
| 40 | 643 |
| 50 | 766 |
| 60 | 867 |
| 70 | 940 |
| 80 | 98˪5 |
| 90 | 100˪0 |

*E. G.* If the Co-latitude at the Middle of the Eclipſe be 60°, and come to be 40° ; ſay, As 867 to 643, So is 19˪1 to 14˪2, which is there to be taken in its ſtead.

*N. B.* The Hint that I had ſeveral Years ago, that in the Determination of Solar Eclipſes the *Equality* of the *Difference* of Sines was made uſe of by Dr. *Halley*, was the Occaſion of the Diſcoveries in theſe Papers.

## XI.

The perpendicular Diſtance of every Point of the Penumbra; and the like Diſtance of every Point of the total Shadow from the Path of the Moon's Center, may be diſcovered by Tables made from the natural Sines; where thoſe Sines themſelves, as before, differ equally, or in arithmetical Progreſſion; according to the Duration of the whole Eclipſe, or of total Darkneſs: and their Co-ſines correſpond to the Diſtances from that Path. Both which Tables here follow:

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*For entire Eclipſes: Which are here ſuppos'd 108'¼ long in the Middle, and the Semi-diameter of the Penumbra 2000 Miles.*

| Duration in Minut. | Diſtance in Miles. | Duration in Minut. | Diſtance in Miles. | | Dig$^{ts}$ |
|---|---|---|---|---|---|
| 1 | 2000 | 37 | 1879 | | |
| 2 | 1999 | 38 | 1872 | | |
| 3 | 1999 | 39 | 1866 | | |
| 4 | 1999 | 40 | 1859 | | |
| 5 | 1998 | 41 | 1852 | | |
| 6 | 1997 | 42 | 1844 | | |
| 7 | 1996 | | | 1838 | 1 |
| 8 | 1995 | 43 | 1835 | | |
| 9 | 1993 | 44 | 1828 | | |
| 10 | 1991 | 55 | 1820 | | |
| 11 | 1990 | 56 | 1812 | | |
| 12 | 1988 | 47 | 1803 | | |
| 13 | 1986 | 48 | 1795 | | |
| 14 | 1984 | 49 | 1786 | | |
| 15 | 1981 | 50 | 1776 | | |
| 16 | 1979 | ½ | 1771 | | |
| 17 | 1976 | 51 | 1766 | | |
| 18 | 1973 | ½ | 1761 | | |
| 19 | 1970 | 52 | 1756 | | |
| 20 | 1966 | ½ | 1751 | | |
| 21 | 1963 | 53 | 1746 | | |
| 22 | 1959 | ½ | 1741 | | |
| 23 | 1955 | 54 | 1736 | | |
| 24 | 1951 | ½ | 1731 | | |
| 25 | 1946 | 55 | 1725 | | |
| 26 | 1942 | ½ | 1720 | | |
| 27 | 1937 | 56 | 1714 | | |
| 28 | 1932 | ½ | 1708 | | |
| 29 | 1927 | 57 | 1703 | | |
| 30 | 1922 | ½ | 1697 | | |
| 31 | 1917 | 58 | 1691 | | |
| 32 | 1911 | ½ | 1685 | | |
| 33 | 1905 | 59 | 1679 | | |
| 34 | 1899 | | | 1676 | 2 |
| 35 | 1893 | ½ | 1672 | | |
| 36 | 1886 | 60 | 1667 | | |

Dura-

| Duration in Minut. | Distance in Miles. | | Dig. |
|---|---|---|---|
| ½ | 1662 | | |
| 61 | 1655 | | |
| ½ | 1648 | | |
| 62 | 1642 | | |
| ½ | 1635 | | |
| 63 | 1628 | | |
| ½ | 1621 | | |
| 64 | 1614 | | |
| ½ | 1609 | | |
| 65 | 1603 | | |
| ½ | 1596 | | |
| 66 | 1589 | | |
| ½ | 1582 | | |
| 67 | 1575 | | |
| ½ | 1568 | | |
| 68 | 1561 | | |
| ½ | 1554 | | |
| 69 | 1546 | | |
| ½ | 1538 | | |
| 70 | 1534 | | |
| ½ | 1528 | | |
| 71 | 1521 | | |
| ½ | 1514 | | |
| | | 1513 | 3 |
| 72 | 1506 | | |
| ½ | 1498 | | |
| 73 | 1490 | | |
| ½ | 1482 | | |
| 74 | 1474 | | |
| ½ | 1456 | | |
| 75 | 1448 | | |
| ½ | 1439 | | |
| 76 | 1430 | | |
| ½ | 1424 | | |
| 77 | 1412 | | |
| ½ | 1403 | | |
| 78 | 1394 | | |
| ½ | 1384 | | |
| 79 | 1374 | | |
| ½ | 1364 | | |

| Duration in Minut. | Distance in Miles. | | Digits |
|---|---|---|---|
| 80 | 1354 | | |
| | | 1350 | 4 |
| ½ | 1344 | | |
| 81 | 1333 | | |
| ½ | 1322 | | |
| 82 | 1312 | | |
| ½ | 1302 | | |
| 83 | 1291 | | |
| ½ | 1280 | | |
| 84 | 1269 | | |
| ½ | 1258 | | |
| 85 | 1246 | | |
| ½ | 1234 | | |
| 86 | 1222 | | |
| ½ | 1110 | | |
| 87 | 1098 | | |
| | | 1188 | 5 |
| ½ | 1186 | | |
| 88 | 1173 | | |
| ½ | 1160 | | |
| 89 | 1147 | | |
| ½ | 1134 | | |
| 90 | 1121 | | |
| ½ | 1107 | | |
| 91 | 1093 | | |
| ½ | 1079 | | |
| 92 | 1065 | | |
| ½ | 1050 | | |
| 93 | 1034 | | |
| | | 1025 | 6 |
| ½ | 1017 | | |
| 94 | 1000 | | |
| ½ | 983 | | |
| 95 | 971 | | |
| ½ | 955 | | |
| 96 | 937 | | |
| ½ | 919 | | |
| 97 | 902 | | |
| ½ | 884 | | |
| 98 | 864 | | |

Dura-

| Duration in Minut. | Diſtance in Miles. | Digts | Duration in Minut. | Diſtance in Miles. | Digts |
|---|---|---|---|---|---|
| | 863 | 7 | ½ | 550 | |
| ½ | 844 | | | 537 | 9 |
| 99 | 824 | | ¾ | 534 | |
| ½ | 804 | | 105 | 517 | |
| 100 | 784 | | ¼ | 499 | |
| ¼ | 773 | | ½ | 482 | |
| ½ | 762 | | ¾ | 463 | |
| ¾ | 751 | | 106 | 443 | |
| 101 | 739 | | ¼ | 421 | |
| ¼ | 728 | | ½ | 399 | |
| ½ | 716 | | ¾ | 376 | |
| ¾ | 704 | | | 375 | 10 |
| | 700 | 8 | 107 | 352 | |
| 102 | 691 | | ¼ | 320 | |
| ¼ | 679 | | ½ | 288 | |
| ½ | 666 | | ¾ | 258 | |
| ¾ | 653 | | 108 | 226 | |
| 103 | 639 | | | 212 | 11 |
| ¼ | 625 | | ¼ | 180 | |
| ½ | 611 | | ½ | 119 | |
| ¾ | 597 | | | 50 | 12 |
| 104 | 582 | | ¾ | 000 | |
| ¼ | 566 | | | | |

## *For Total Darkneſs of* 166″ ⅔.

| Duration in Secds. | Diſtance in Miles. | Duration in Secds. | Diſtance in Miles. |
|---|---|---|---|
| ″ | | ″ | |
| 1 | 50,0 | 37 | 49,0 |
| 2 | 50,0 | 38 | 49,0 |
| 3 | 50,0 | 39 | 49,0 |
| 4 | 50,0 | 40 | 48,9 |
| 5 | 50,0 | 41 | 48,9 |
| 6 | 50,0 | 42 | 48,8 |
| 7 | 50,0 | 43 | 48,7 |
| 8 | 50,9 | 44 | 48,6 |
| 9 | 50,9 | 45 | 48,5 |
| 10 | 50,9 | 46 | 48,4 |
| 11 | 49,9 | 47 | 48,2 |
| 12 | 49,8 | 48 | 48,1 |
| 13 | 49,8 | 49 | 47,9 |
| 14 | 49,8 | 50 | 47,7 |
| 15 | 49,7 | 51 | 47,6 |
| 16 | 49,7 | 52 | 47,5 |
| 17 | 49,7 | 53 | 47,4 |
| 18 | 49,6 | 54 | 47,3 |
| 19 | 49,6 | 55 | 47,2 |
| 20 | 49,6 | 56 | 47,1 |
| 21 | 49,6 | 57 | 47,0 |
| 22 | 49,5 | 58 | 46,9 |
| 23 | 49,5 | 59 | 46,8 |
| 24 | 49,5 | 60 | 46,6 |
| 25 | 49,4 | 61 | 46,5 |
| 26 | 49,4 | 62 | 46,4 |
| 27 | 49,4 | 63 | 46,3 |
| 28 | 49,3 | 64 | 46,2 |
| 29 | 49,3 | 65 | 46,0 |
| 30 | 49,3 | 66 | 45,9 |
| 31 | 49,2 | 67 | 45,8 |
| 32 | 49,2 | 68 | 45,6 |
| 33 | 49,2 | 69 | 45,5 |
| 34 | 49,1 | 70 | 45,3 |
| 35 | 49,1 | 71 | 45,2 |
| 36 | 49,1 | 72 | 45,1 |

Dura-

| Duration in Sec^ds^. | Diſtance in Miles. | Duration in Sec^ds^. | Diſtance in Miles. |
|---|---|---|---|
| ″ | | ″ | |
| 73 | 45,0 | 111 | 37,4 |
| 74 | 44,8 | 112 | 37,1 |
| 75 | 44,6 | 113 | 36,8 |
| 76 | 44,5 | 114 | 36,5 |
| 77 | 44,3 | 115 | 36,2 |
| 78 | 44,1 | 116 | 35,9 |
| 79 | 44,0 | 117 | 35,6 |
| 80 | 43,8 | 118 | 35,3 |
| 81 | 43,7 | 119 | 35,0 |
| 82 | 43,5 | 120 | 34,7 |
| 83 | 43,3 | 121 | 34,4 |
| 84 | 43,2 | 122 | 34,1 |
| 85 | 43,0 | 123 | 33,8 |
| 86 | 42,9 | 124 | 33,4 |
| 87 | 42,7 | 125 | 33,0 |
| 88 | 42,5 | 126 | 32,7 |
| 89 | 42,3 | 127 | 32,4 |
| 90 | 42,1 | 128 | 32,1 |
| 91 | 41,9 | 129 | 31,7 |
| 92 | 41,7 | 130 | 31,3 |
| 93 | 41,5 | 131 | 30,9 |
| 94 | 41,3 | 132 | 30,5 |
| 95 | 41,1 | 133 | 30,1 |
| 96 | 40,9 | 134 | 29,7 |
| 97 | 40,7 | 135 | 29,3 |
| 98 | 40,5 | 136 | 28,9 |
| 99 | 40,3 | 137 | 28,5 |
| 100 | 40,0 | 138 | 28,0 |
| 101 | 39,8 | 139 | 27,5 |
| 102 | 39,6 | 140 | 27,1 |
| 103 | 39,4 | 141 | 26,6 |
| 104 | 39,1 | 142 | 26,1 |
| 105 | 38,8 | 143 | 25,6 |
| 106 | 38,6 | 144 | 25,1 |
| 107 | 38,4 | 145 | 24,6 |
| 108 | 38,1 | 146 | 24,0 |
| 109 | 37,9 | 147 | 23,5 |
| 110 | 37,6 | 148 | 23,0 |

| Duration in Sec$^{ds}$. | Diſtance in Miles. | Duration in Sec$^{ds}$. | Diſtance in Miles. |
|---|---|---|---|
| ″ |  | ″ |  |
| 149 | 22,4 | 159 | 14,9 |
| 150 | 21,8 | 160 | 14,0 |
| 151 | 21,1 | 161 | 12,9 |
| 152 | 20,5 | 162 | 11,7 |
| 153 | 19,8 | 163 | 10,4 |
| 154 | 19,1 | 164 | 8,9 |
| 155 | 18,3 | 165 | 7,0 |
| 156 | 17,6 | 166 | 4,4 |
| 157 | 16,7 | 166 $\frac{2}{3}$ | 0,0 |
| 158 | 15,9 |  |  |

Semidiameter of the Umbra or Total *D*arkneſs = 50 Miles.

## XII.

The Angles at the Vertex of the Lunar Circle, on each Side of the Point of Contact, by Reaſon of the perpendicular Situation of that Axis to its own Circle; are always right Angles: Only diminiſh'd in the Proportion of the Minutes deſcrib'd by the Annual Motion during the Continuance of the Eclipſe. Thus in our preſent Eclipſe, which retains the Center of the Shadow near three Hours upon the Earth's Diſk, in which Time the annual Motion amounts to about 8′; each of thoſe right Angles in Strictneſs are to be eſteem'd only 89° 56′, and both together 179° 52′. Only becauſe the Refraction of the Rays of the Sun through our Atmoſphere, requires a ſomewhat greater Increaſe of this Angle, than the annual Motion requires its Diminution, I ſhall wholly omit it, in all my Calculations hereafter.

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## XIII.

The Angles made at the Poles of the Earth, which ſhew the Difference of the two extreme Meridians, and limit the Extent of the entire central Eclipſe, by reaſon of the Obliquity of the Earth's Axis to that Lunar Circle, are uſually unequal to one another; and more or fewer than twice 90°, as the Eclipſe happens at different Latitudes of the Moon, and Times of the Year.

## XIV.

The Meridian that paſſes through the Middle or Central Point of general Solar Eclipſes, is the ſame with that which paſſes through the Center of the Sun at the ſame Time, when the Moon has no Latitude, and the Eclipſes are Central in the Plane of the Eclíptick; as alſo when they happen in either Solſtices. Otherwiſe the Moon's Latitude being taken perpendicular to the Plane of the Ecliptick; and the neareſt Diſtance of the Moon's Motion being taken perpendicular to the Lunar Circle, while the Meridians always paſs through the Poles of the Earth; theſe Two Meridians will, generally ſpeaking, be different, and their included Angle no otherwiſe to be known than by Trigonometry, as will appear hereafter.

## XV.

The Dimenſions of the Penumbra, or entire Eclipſe, and the Extent of the total Shadow on the Earth, are continually different, according to the different Elevations of the Sun and Moon above any particular Horizon. For as the Moon is about the ſame Diſtance from every Place, when it is in its Horizon, as it is from the Earth's Center it ſelf; with regard to which Center alone our firſt Calculations are always made: So when it is in the Zenith of any Place, it is one Semidiameter of the Earth nearer it; which Semidiameter being uſually $\frac{1}{60}$, and at our next Eclipſe $\frac{1}{55\llcorner 3}$ of its entire Diſtance, as will appear hereafter, will deſerve an Allowance. Nor will any leſſer Elevation of the Moon be wholly inconſiderable in Eclipſes, but in all accurate Determinations thereof muſt be particularly computed, in order to the diſtinct Knowledge of the Extent of ſuch Eclipſes; eſpecially of the Breadth of the Total Shadow therein. Accordingly we are to obſerve that this Breadth of the Total Shadow will certainly be at this Eclipſe conſiderably greater over *North America*; where the Luminaries are greater elevated above the Horizon; than over *Europe*, where they are much nearer it; as this Calculation requires: Of which hereafter.

## XVI.

The *Figure* of the entire *Penumbra*, or general Eclipſe; and of the *Umbra*, or Total Darkneſs; as they appear upon every Country, is different, on account of the different Obliquity of every Horizon; and will make Ovals or Ellipſes of different Species perpetually. This in the vaſt *Penumbra* is beſt underſtood by ſuch an Inſtrument as my *Copernicus*; or by the Peruſal of a very ſcarce Book written by P. *Courſier*, (*Philoſ. Tranſact*. N°. 343. p. 259.) and cited by Dr. *Halley*: Which diſtinctly treats of the Interſection of a Conical and Spherical Surface. But in the ſmaller *Umbra*, or Total Darkneſs, which is confined to a much narrower Compaſs, it very nearly approaches to the Interſection of a conick Surface with a Plane, which is a true Ellipſis.

## XVII.

The *Species* of that Ellipſis depends on the Sun's Altitude above the Horizon at the Time of Total Darkneſs; as does the *Poſition* of its longer Axis on the Azimuth of the Sun at the ſame Time. Nor is it at all neceſſary that the Direction of this or any other Ellipſis ſhould be along either of the Axes, but may as well be along any other Diameter whatſoever.

## XVIII.

The *Direction* of the Center of the Shadow is according to the Direction of the Moon's Motion, along the Plane of the Lunar Circle, as compounded with the diurnal Motion, or with the Direction and Velocity of thoſe Parts of the Earth over which it paſſes; and will be hereafter brought to Calculation. And indeed this Angle may be had, either by finding the ſeveral Points of the Path of the Moon's Way upon the Earth, in as many Meridians as we pleaſe, and drawing a curve Line through thoſe Points; or by ſolving a Spherical Triangle, whoſe Sides are the Complements of the Latitudes of two neighbouring Places equally diſtant, *Eaſt* and *Weſt*, from the Place you work for; and whoſe included Angle is the Angle at the Pole ſuited to the Difference of their Meridians; and taking half the the two Angles at the Baſe, the one internal and the other external, for the Angle deſir'd: Of which hereafter.

## XIX.

Every Ellipſis, made by the oblique Section of a Cone, has the Interſection of the (*Fig.* 1.) Axis of the Cone C at ſome Diſtance from the Center of the Ellipſis *D*. And the Proportion of thoſe unequal Diviſions *B C* and *C A* are the ſame with that of the Sides of the Cone *V B* and *V A* : As appears by the (*Elem.* III. 6.) Elements of *Euclid.* Whence it is evident, that the proper Center of the total Shadow in Eclipſes of the Sun, or that made by the Axis of the Cone, is not the ſame with the Center of the Elliptick Shadow; and that the Proportion of its Diſtance from that Center may be eaſily determin'd by the Propoſition here refer'd to: Of which more hereafter.

*Scholium.* This Ellipſis, when the Sun is of a conſiderable Altitude, is almoſt an exact one; but when the Sun is near the Horizon, it will be very long, and ſo leſs exact; becauſe the Spherical Surface of the Earth is at that Diſtance more remote from a Plane.

## XX.

The perpendicular *Breadth* of the Shadow is neither that of the longer, nor that of the ſhorter Axis: But that of the two longeſt Perpendiculars (*Fig.* 2.) *A B* and *CD* drawn from the Tangents parallel to the Diameter *D B*, along which the Direction of the Motion is: The length of which Perpendiculars will be hereafter determined.

## XXI.

The *Velocity* of the Motion of the Center of the Shadow is unequal; not only on account of the Difference of the Moon's own Motion, at the beginning and ending of the entire Eclipſe; which indeed is very inconſiderable; but chiefly by reaſon of the Difference of the Obliquity of the Horizon all the Way of its Paſſage. However, ſince the ſeveral Points may, in all Meridians be diſtinctly found by Trigonometry, as we ſhall ſhew preſently, this Inequality need create us no new Difficulty in the Determination of Eclipſes.

## XXII.

The Number of *Digits* eclips'd, which are twelfth Parts of the Sun's Diameter, with ſexageſimal Parts of the ſame Digits, are always to be eſtimated as diſtinct from the total Shadow; and may be diſcovered by the help of the foregoing Table, p. 29, 30, 31. Where the Digits are already noted at every proper Diſtance from the Path of the Moons Center; and where the intermediate Fraction $\frac{1}{162\llcorner 6}$ is more exact than $\frac{1}{27}$; but which, by dividing that Number $\frac{1}{162\llcorner 6}$ by $2\llcorner 7$ will give thoſe Sexageſimals, without any farther Trouble. The Application of that Table will be taught hereafter.

## XXIII.

The *Diſtance* of the Vertex of the conical Shadow of the Moon, which ſometimes juſt reaches the Surface of the Earth, as in total Eclipſes *ſine morâ*, Sometimes does not reach it; as in Annular Eclipſes; and ſometimes would overreach it, if it were not intercepted, as in total Eclipſes *cum morâ*; may be eaſily diſcovered at any time by the Analogy following: As $PC$ the (*Fig.* 3.) Semidiameter of the Moon: $= 941$ geographical Miles is to $CV$ the Diſtance of the Moon from that Vertex $= 215000$ :: So is $Rs = 45$ the ſmalleſt Semidiameter of the total Shadow, which is the ſame as of the circular Shadow it ſelf, to $sV$, the Diſtance of that Vertex there-from = 10316, which is thus: 941 : 215000 :: 45 : 10316.

## XXIV.

The Determination of the Circumſtances of Solar Eclipſes, for any given Diſtance from the Path of the Moon's Center, either way, has no new Difficulty in it; but is to be made juſt as is that for the Center of the Penumbra. Only the Quantity of the Diſtance of the Vertex of the Lunar Plane from that Circle will be different; as the Path of the Moon's Center it ſelf might be at another Eclipſe, of otherwiſe the ſame Circumſtances.

## XXV.

If Two Bodies *A* and *B* ſet out together, the one from *A*, the other from *B*: and move evenly forward in a known Proportion as to Velocity; the Point C will be determined where the ſwifter will overtake the ſlower, and they will be coincident. Thus if the Velocity of *A*, (*Fig*. 4.) be to that of *B*, as 5 to 1, the Proportion of the Lines *A B* to *B C* will be as 4 to 1, and if we add 1 to 4 = 5 we have the place *C* where the ſwifter will overtake the ſlower. Thus if their Velocities be to each other as 5⌞48 to 1, the Lines of their Motion *A B* and *B* C, will be as 4⌞48 to 1. So that if we take in the former Caſe $\frac{1}{4}$ of *A B* and in the other $\frac{1}{4\llcorner 48}$ of *A B*, and add it to *A B*, we gain *AC* the Diſtance of the Point C from *A*.

*Corollary*. If therefore *A* repreſent the Center of the Shadow of a Solar Eclipſe, as it is plac'd at the Middle of the general Eclipſe; and B *Greenwich* at the ſame Moment of abſolute Time; and at a known Diſtance from the middle

Point; and if the Velocity of the Center of the Shadow along the Circle be to the Velocity of *Greenwich* in its diurnal Motion as 5,48 to 1, if we ad 4,48 of their Diſtance at their ſetting out to that known Diſtance, we obtain the Point or Place where the Center of the Eclipſe will overtake *Greenwich*, or the Time when the Eclipſe will be at the Meridian of *Greenwich*. And this whether the Center and *Greenwich* move along the ſame Line as *ABC*, or two different Lines, as *ABC* and *abc*.

## XXVI.

The Duration of Solar Eclipſes is different, according as their Middle happens about Six in the Morning or Evening; or about Noon; or about any intermediate Time. If that happens about Six a-Clock, Morning or Evening, the diurnal Motion then neither much conſpires with, nor oppoſes the proper Motion of the Center of the Shadow; and the Duration is almoſt the ſame as it would be if the Earth had no diurnal Motion at all. If that happens about Noon, the diurnal Motion moſt of all conſpires with that proper Motion of the Center, and makes the Duration of the Eclipſe the longeſt poſſible. If it happens in the intermediate Times, the diurnal Motion, in a leſs Degree, conſpires with the other Motion, and makes the Duration of a mean Quantity, between that of the other Caſes. But if it happens conſiderably before Six a-Clock in the Morning, or after Six a-Clock in the Evening, the diurnal Motion is backward, and ſhortens that Duration proportionably. Of the Quantity of which Duration we ſhall enquire more hereafter.

# PROBLEMS.

## I.

To find the neareſt Diſtance of the Path of the Moon's Center, to the Center of the Diſk of the Earth, as ſeen at the Diſtance of the Moon in the total Eclipſe of the Sun *A. D.* 1724. *May* 11°. *P. M.*

This is equal to the Moon's true Latitude at the Time of the Conjunction in her own Orbit; and is ſet down in the Calculation 32′ 19″.

## II.

To find the Sun's *Declination* at the Middle of the Eclipſe.

As the Radius of the Circle: is to the Sine of the Sun's Longitude at that Time=61′ 39″: : So is the Sine of the Sun's greateſt Declination, = 23° 29′: to the Sine of the Sun's Declination then.

Rad. 10. 000000
S. 61° 39′ 9. 944514
S. 23° 29′ 9. 600409 ° ″
S. 18. 544923 = 20 32
= Declination ☉.

## III.

To find the Sun's *Right Aſcenſion* for the ſame Time.

As the Radius : to the Coſine of the Sun's greateſt Declination :: So is the Tangent of the Sun's Latitude: to the Tangent of the Sun's right Aſcenſion.

Rad. 10.
Sin. 66 31 9. 962453
Tang. 61 39 10. 267952 ° ′
Tang. Right Aſcen. 10. 230405 = 59 31
Compl. 30 29

## IV.

To find the *Diſtance* between the Vertex of the Lunar Circle and the Pole of the Earth.

Let (*Fig.* 5.) *E P* repreſent the Diſtance between the two Poles of the Earth, and of the Ecliptick : = 23° 29′. *EV* the Diſtance between the Pole of the Ecliptick, and the Vertex of the Lunar Circle : = 5° 36′. *P E V* the Angle made by the Solſtitial Colure *P E* : (in which the two Poles of the Earth, and of the Ecliptick always are) and that Arc *E V*. And becauſe the Sun is here 28° 21′ diſtant from that Colure, which is the Complement of its Longitude from *Aries* ; and the Aſcending Node, or Argument of Latitude is then 5° 49′ diſtant from the Sun backward ; The Sum of theſe Numbers gives 34° 10′, whoſe Complement is the Angle *V E R* = 55° 50′. In order then to gain *V P* proceed thus :

R. 10.
CS *V E R*. 55 50. 9. 749429
T. *V E*. 5 36 8. 991451 ° ′
T. *R E* 18. 740880 = 3. 10.
+ 23. 29.
= 26. 39.

Then ſay, ° ′

CS. *R E* 3 9 9. 992343
CS. *V E* 5 36 9. 997922
CS. *P R* 26 39 9. 951222
19. 949144 ° ′
CS. *P V* = 9. 949891 = 27. 0

V. In

## V.

In the ſame Triangle *VPE*, to find the Angle *VPE*, included between the Colure *EP*, and the Prime Meridian *PV*.

° ′
S. *PV*. 27 0. 9. 657047
S. *VER* 55 50. 9. 917719
S. *VE* 5 36. 8. 989374
18. 907098 ° ′
S *VPE* = 9. 249961 = 10 15.

*Corollary.* The *Primary Angle*, compoſed of the Complement of the Right Aſcenſion, and of this Angle *VPE*, is = 40° 44′.

Compl. Rad. 30° 29′
+ 10 15
= 40 44

## VI.

To find the *Diſtance* of the Pole or Vertex of the Lunar Circle from the Circle it ſelf.

As the Semidiameter of the Earth's Diſk = 61′ 38″: To the Latitude of the Moon, or neareſt Diſtance to the Path of the Moon's Center from the Center of the Diſk; 32° 19′ :: So is the Radius: to the Sine of the Complement of that Diſtance. In Decimals thus: 61′,63 : 32′,32 :: 10000 : 5244 = S. 31° 38′, whoſe Complement is 58° 22′: equal to the Diſtance of the Lunar Circle from its Vertex.

## VII.

To find the *Angle* included between the Meridian that paſſes through the Center of the general Eclipſe, and that paſſing through the Center of the Sun at the ſame Time.

In the Triangle (*Fig.* 6.) *MSP* where the Side *SP* is already given, = Complement of the Sun's Declination; find the Angle *MSP* thus:

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R. 10.
S. of the Sun's Diſtance from the Solſtitial Colure: = 28° 21′ 9. 676562
S. of the Sun's greateſt Declination = 23° 29′ 9. 600409 ° ′
S. of an Angle $^{1}$9. 276971 = 10. 54.
To which add the Angle of the Moon's Way: + 5 36
The Sum is the Angle *MSP* = 16. 30

Then, in the ſame Triangle *PMS*, we have two Sides: *MS* equal to the Latitude of the Moon, or the length of the Perpendicular to the Moon's way = 31° 38′. and *PS* = 69° 28′. and the included Angle *MSP* = 16° 30′, to find *MPS* thus:

R. 10.
° ′
CS. *MSP*. 16. 30. 9. 981774
T. *SM*. 31. 38. 9. 789585
T. *RM*. = $^{1}$9. 771359 = 30 34 *MR*
+ 31 38 *MS*
Then ſay, = 62 12 *RS*
S. *SP*. 69. 28. 9. 971493
S. *SR*. 62. 12. 9. 946738
T. *MSP*. 16. 30. 9. 471605
$^{1}$9. 418343 ° ′
T. *MPS* = 9. 446850 = 15 38
+ 40 44
= 56 22 = Angle with the Primary Meridian.

## VIII.

### To find the *Longitude* and *Latitude* of the Center of the Shadow at the Middle of the general Eclipſe; or to ſolve the primary Triangle.

Under *Problem* IV. we have found the Diſtance of the Vertex of the Lunar Circle from the Pole of the Earth = 27°. 0′. Under *Problem* VI. we have found the Diſtance of that Vertex from its Circle = 58°. 22′; and under the laſt *Problem* we have found the Angle at the Pole of the Earth, between the Primary Meridian, and that Meridian which paſſes through the Center of the Eclipſe, = 56° 22′. From which *data* the Primary Triangle (*Fig.* 7.) is thus to be ſolv'd:

R. 10.
° ′
CS. *CPQ*. 56. 22. 9. 743412
T. *VP*. 27. 0. 9. 707166
T. *P R*. = 15. 46. 19. 450578

Then ſay,

CS. *VP* 27. 0. 9. 949880
CS. *PR* 15. 46. 9. 983345
CS. *VC*. 58. 22. 9. 719730
19. 703075 ° ′
CS. *PC*. 9. 753195 = 55. 30
Deduct *PR* = 15. 46
Remt. = *PC*. = 39. 44
*Ergo* Lat. = 50. 16

To find the Angle at the Vertex *CVP*, proceed in this Manner:

S. *CV*.

| | ° | ′ | | ° | ′ |
|---|---|---|---|---|---|
| S. *CV.* | 58. | 22. | 9. 930145 | | |
| S. *CPQ.* | 56. | 22. | 9. 920436 | | |
| S. *PC.* | 39. | 44. | 9. 805647 | | |
| | | | 19. 726083 | | |
| S. *CVP.* = | | | 9. 795938 = | 38 | 41 |

The following Analogy will give *VCP* the Complement of the Angle which the Direction of the Center of the Eclipſe makes with the Meridian, that Direction being perpendicular to *VC.*

| | ° | ′ | | ° | ′ |
|---|---|---|---|---|---|
| S. *CV.* | 58. | 22. | 9. 930145 | | |
| S. *CPQ.* | 56. | 22. | 9. 920436 | | |
| S. *VP.* | 27. | 0. | 9. 657047 | | |
| | | | 19. 577483 | | |
| S. *VCP.* = | | | 9. 647338 = | 26. | 21 |
| | | | Compl. = | 63. | 39 |

*Corollary.* Hence we alſo learn the moſt *Northern Latitude*, where the Center of the Shadow will croſs the Meridian at Noon, and at right Angles: And this without any particular diſtinct Calculation. For *VQ* = 58° 22′ — *VP* 27° 0′ = *PQ* = 31° 22′. whoſe Complement = 58° 38′. is that very *Northern Latitude.*

## IX.

To find the *Longitude* and *Latitude* of the Center of the Shadow, when it croſſes the Meridian that paſſes through the Center of the Sun at the Middle of the Eclipſe; or to ſolve the ſecond principal Triangle.

If in the foregoing Triangle we ſuppoſe the Angle at the Pole to be equal to the Primary Angle, or 40° 44′. we may thus ſolve this Triangle:

R. 10.
° ′
CS. *CP*𝒬. 40. 44. 9. 879529
T. *VP*. 27. 0. 9. 707166 ° ′
T. *PR*. = 19. 586695 = 21 7

Then ſay,

CS. *VP*. 27. 0. 9. 949880
CS. *PR*. 21. 7. 9. 969811
CS. *VC*. 58. 22. 9. 719730
19. 689541 ° ′
CS. *CR*. = 9. 739661 = 56 42
Deduct 21 7
Remᵗ. *CP*. 35 35
*Ergo*, Lat. = 54 25

To find the Angle at the Center of the Eclipſe *VCP*, proceed thus:

° ′
S. *VC*. 58. 22. 9. 930145
S. *CP*𝒬 40. 44. 9. 814607
S. *VP*. 27. 0. 9. 657047
19. 471654 ° ′
S. *CVP*. = 9. 541509 = 20 22

## X.

To find the *Longitude* and *Latitude* of the Center of the Shadow at its Entrance on the Diſk of the Earth: Or to ſolve the third Principal Triangle.

Add the vertical Angle already found = 38° 41′ to a right Angle at the Vertex; = 90 + 38°. 41′ = 128°. 41′ this is equal to the Angle at the Vertex *CVP*. Subſtract this Angle from two right Angles. 180° — 128°. 41′ = 51°. 19′, in order to gain the Supplement, whoſe Sines, *&c.* are the ſame with the others. (*Fig.* 8.) Then ſay,

R. 10.

° ′

CS. *CVP*. 51. 19. 9. 795891
T. *VP*. 27. 0. 9. 707166 ° ′
T. *VR*. = 19. 503057 = 17 40 = *VR*
+ 58 22 = *VC*

Then ſay, = 76 02 = *CR*

° ′

CS. *VR*. 17. 40. 9. 979019
CS. *VP*. 27. 00. 9. 949880
CS. *CR*. 76. 02. 9. 382661
19. 332541 ° ′
CS. *CP*. = 9. 353522 = 76 : 57
*Ergo*, Lat. 13. 03

In order to find the Angle at the Pole *VPC*, whoſe Supplement is the Longitude of that Point where the Center of the Shadow enters the Diſk of the Earth from the Primary Meridian, proceed thus:

° ′

S. *CP*. 76. 57. 9. 988636
S. *CVP*. 51. 19. 9. 892435
S. *CV*. 58. 22. 9. 930145
19. 822580 180 °
S. *VPC*. = 9. 833944 = 43 01
Suppl. 136 : 59

To find the Angle *VCP*, proceed thus:

° ′
S. *CP*. 76. 57. 9. 988636
S. *CVP*. 51. 19. 9. 892435
S. *VP*. 27. 00. 9. 657047
19. 549482 ° ′
S. *VCP*. = 9. 560846 = 21. 20

## XI.

To find the Longitude and Latitude of the Center of the Shadow at its Exit from the Diſk of the Earth; or to ſolve the fourth Principal Triangle.

Subſtract the Angle already found 38° 41′ from a right Angle 90° — 38° 41′ = 51° 19 = Angle at the Vertex *PVC* (*Fig.* 9.) Then ſay,

R. 10.
° ′
CS. *PVC*. 51. 19. 9. 795891
T. *PV*. 27. 00. 9. 707166 ° ′
T. *VR*. 9. 503057 = 17 40
from 58 22
Then ſay, Remnt 40 42 = *RC*.
° ′
CS. *VR*. 17. 40. 9. 979019
CS. *PV*. 27. 00. 9. 949880
CS. *RC*. 40. 42. 9. 879746
19. 829626 ° ′
CS. *PC* = 9. 850607 = 44. 51.
*Ergo*, Lat. 45. 9.

In order to find the Angle *QPC*, or the Longitude of that Point where the Center of the Shadow departs out of the Diſk of the Earth, from the Primary Meridian, proceed thus:

° ′
S. *CP*. 44. 51. 9. 848345
S. *PVC*. 51. 19. 9. 892435
S. *VC*. 58. 22. 9. 930145
19. 822580 ° ′
S. *QPC* = 9. 974235 = 70. 27
+ 136 59
= 207 26

And for the Angle *VCP* thus;

° ′
S. *CP*. 44. 51. 9. 848345
S. *PVC*. 51. 19. 9. 892435
S. *VP*. 27. 00. 9. 657047
19. 549482 ° ′
S. *VCP*. = 9. 701137 = 30. 10.

*Corollary.* Hence the Angular Motion of the Center of the Eclipſe about the Pole of the Earth, if there were no diurnal Motion, is 207°. 26′.

## XII.

To find the Time in which the Center of the Shadow will go over the Diameter of the Lunar Circle.

Say, firſt, 35′⌊3 : 60′ :: 123⌊26 : 209⌊6 ; *i. e.* As the Number of Minutes of a Degree paſs'd over in an Hour : to an Hour : : So is the entire Diameter of the Diſk from the Calculation : to the Number of Minutes for that Paſſage.

Then ſay, 10000 : 8514 : : 209′⌊6 : 178′⌊4 *i. e.* As the Radius : to the Sine of 58° : 22′ = the Diſtance of the Lunar Circle from its Pole or Vertex : : So are the Minutes of the Paſſage over the entire Diameter : to the Minutes of the Paſſage over this Chord = 178′ 24″.

## XIII.

To find the Proportion of the Velocities of the Center of the Shadow and of the diurnal Motion of the correſponding Point of the Earth at the Time of the Eclipſe:

Say thus; As 178′⌞4 to 829′⌞6 = 207° 26′, or as 1 to 4⌞64, ſo is the Time of the Center of the Eclipſe's Motion over the Diameter of the Lunar Circle: to the Timeof the diurnal Motion's going from the entranceto the Exit of the Center.

*Corollary*. Hence the real Angular Motion of the Center of the Eclipſe about the Pole of the Earth, is no more than 162° 40′. For 4⌞64: 3⌞64 :: 207° 26′ : 162° 40′.

## XIV.

To find the *Latitude* of any Place, over or near which the Center of any Shadow paſſes, to any known *Longitude* or *Time* given. And, *vice verſa*, To find the *Longitude* or *Time* of the neareſt Approach to any ſuch Place to any known *Latitude*. This is no more than proceeding in the Calculations as hitherto; by taking any known Meridian or Time; or elſe any known Latitude for our Examples.

I ſhall therefore give three ſeveral Examples in both Caſes; becauſe of the great Dignity and Uſefulneſs of the *Problem*: *viz.* For *Greenwich* the Meridian of the Tables; for *Dublin* more *Weſtward*; and for *Paris* more *Eaſtward*.

Now I here ſuppoſe, from the Calculation and Conſtruction of Eclipſes, that the Middle of this general Eclipſe will happen *May* 11. 1724. 17′ paſt 5 a-Clock in the Afternoon; and that its Center will croſs the Meridian of *Greenwich* 41′ paſt 6. Upon which Hypotheſis I thus compute:

From the Angle 100° 15′ = $6^{h}$ 41′
Deduct the Primary Angle 40 44
There remains the Angle of the Pole *QPC*=59 31

Then proceed thus:

R. 10.
CS. *QPC.* 59° 31′ 9. 705254
T. *PV.* 27 00 9. 707166
T. *PR.* = 19. 412420 = 14° 29′

Say then,

CS. *PV.* 27 00 9. 949880
CS. *PR.* 14 29 9. 985974
CS. *VC.* 58 22 9. 719730
19. 705704
CS. *RC.* = 9. 755824 = 55° 15′
Deduct *RP* = 14 29
Remains *PC* =40 46
*Ergo*, Lat. 49 14

*N. B.* If we take Dr. *Halley*'s Time $6^{h}$ 36′, and ſubſtract 40° 44′ out of 99° there remains 58° 16′; and the Calculation will ſtand thus:

R. 10.
CS. *QPC.* 58°. 16′. 9. 720958
T. *PV.* 27. 0. 9. 707166
T. *PR.* = 19. 428124 = 15° 0′

Then ſay, ° ′
CS. *PV.* 27. 0. 9. 949880
CS. *PR.* 15. 0. 9. 984944
CS. *VC.* 58. 22. 9. 719730
19. 704674 ° ′
CS. *RC.* = 9. 754794 = 55 21
Deduct *RP.* 15 0
Rem$^{t}$. *PC.* 40 21
*Ergo*, Lat. = 49 39
Diff. 00 25

*N. B.* My Calculation differs from Dr. *Halley*'s Scheme no leſs than a full Degree of a great Circle, in the Meridian; if our Difference of Time, which is about 5′, be allowed. And though we take the Doctor's own Time, yet do we differ in *Latitude* 25 Minutes or Miles; by which Quantity the Doctor's Scheme brings the Center of the Eclipſe nearer to *London* and *Greenwich* than this Calculation. The reaſon of which Difference I by no means underſtand. Time will diſcover which Determination is moſt accurate.

*Dublin* is about 6° 22′ *Weſtward* in Longitude from *Greenwich.* Let us find the Latitude of the Center of the Shadow, when it croſſes the Meridian of *Dublin.* We muſt proceed thus:

As 3,64 to 4,64 ſo is 6′. 22″ to 8′. 7″ Deduct then from the Angle at the Pole uſed for *Greenwich* this Difference of theſe Angles 59° 31′ — 8°. 7′ = 51° 24′. which is our Angle at the Pole for *Dublin.* So that if we uſe the former Figure with that Angle, we compute as before;

R. 10.
° ′
CS. *QPC.* 51. 24. 9. 795101
T. *PV.* 27. 0. 9. 707166 ° ′
T. *RP.* 19. 502267 = 17. 38

Then ſay,

| | ° | ′ | |
|---|---|---|---|
| CS. *PV.* | 27. | 0. | 9. 949880 |
| CS. *R P.* | 17. | 38. | 9. 979099 |
| CS. *V* C. | 58. | 22. | 9. 719730 |
| | | | 19. 698829 |

| | | ° | ′ |
|---|---|---|---|
| CS. *R* C. = | 9. 748949 = | 55 | 53 |
| Deduct *R P.* | | 17 | 38 |
| Remn^t P C. | | 38 | 15 |
| *Ergo*, Lat. | | 51 | 45 |

*Paris* is 2° 19′ more *Eaſterly* than *Greenwich*,
Say therefore $3_L64 : 4_L64 :: 2^{\circ}.\ 19' : 2^{\circ}.\ 57'$
Now $59^{\circ}.\ 31' + 2^{\circ}.\ 57' = 62^{\circ}.\ 28'$

Then, R. 10.

| | ° | ′ | |
|---|---|---|---|
| CS. *Q P C.* | 62. | 28. | 9. 664891 |
| T. *P V.* | 27. | 0. | 9. 707166 |

| | | ° | ′ |
|---|---|---|---|
| T. *R P.* = | 19. 372057 = | 13 | 15 |

Say then,

| | ° | ′ | |
|---|---|---|---|
| CS. *P V* | 27. | 0. | 9. 949880 |
| CS. *R P* | 13. | 15. | 9. 988282 |
| CS. *V* C. | 58. | 22. | 9. 719730 |
| | | | 19. 708012 |

| | | ° | ′ |
|---|---|---|---|
| CS. *R* C. | 9. 758132 = | 55. | 03 |
| Deduct *R P* = | | 13. | 15 |
| Rem^t. = P C. | | 41. | 48 |
| *Ergo*, Lat. = | | 48. | 12 |

*N. B.* By ſuch Calculations we may determine the Latitude of the Center of the Shadow's Way, from its entry upon, till its exit out of the Diſk of the Earth, to every known Meridian. A Specimen of which I intend to give preſently for the ſeveral *Eaſt* and *Weſt Longitudes* from *London* in the Eclipſe before us: And another Specimen in the Eclipſe, *Sept.* 4. 1727.

If the *Latitudes* be given, as for the Meridian of *Greenwich* 49°. 14′; For *Dublin* 51°. 45′; For *Paris* 48°. 12′: The Caſe will be that of a Spherical Triangle, when all the Sides are given; and the *Longitude* or *Time* is an Angle ſought. Thus in the foregoing Figure for *Greenwich*, *VC* = 58°. 22′ is the Side againſt the Angle ſought. *VP* = 27°. 0′, and *PC* = 40°. 46′. From which *data* we thus diſcover the Angle *QCP*.

| | ° | ′ | | |
|---|---|---|---|---|
| *VC* = | 58 | 22 | R. | 10. |
| *VP* = | 27 | 00 | S. | 9. 657047 |
| *PC* = | 40 | 46 | S. | 9. 814900 |
| Sum of 3 = | 126 | 08 | Sum | 19. 471947 |
| ½ Sum | 63 | 4 | S. | 9. 950138 |
| Diff. of *VC* | 4 | 42 | S. | 8. 913488 |
| Double Radius | | | | 20. |
| The Sum | | | | 38. 863626 |
| The Remainder | | | | 19. 391679 |
| ½ Remainder | | | | 9. 695839 = CS. 60° 14′ |

double 120. 28
from 180. 00
as before remains 59. 32
Add the Primary Angle 40. 44
Sum 100. 16
Equal in Time to $6^{h}$. 41′

The Time paſt Noon of the Center's Tranſit over the Meridian of *Greenwich*.

For *Dublin* thus:

| | ° | ′ | | | |
|---|---|---|---|---|---|
| *V C* = | 58 | 22 | R. | 10. | |
| *V* P | 27 | 00 | S. | 9. 657047 | |
| P *C* | 38 | 15 | S. | 9. 791756 | |
| Sum of 3. | 123 | 37 | Sum | 19. 448803 | |
| Half Sum | 61 | 48 ½ | | 9. 945159 | |
| Diff. of *V C* | 3 | 26 ½ | | 8. 778383 | |
| Double Radius | | | | 20. | |
| The Sum | | | | 38. 723542 | |
| The Remainder | | | | 19. 274739 | |
| Half the Remainder | | | | 9. 637369 | C S. = $64^{\circ}\ 18'$ |

double 128. 36
from 180. 00
remains as before 51. 24.
To 51. 24.
Add 40. 44.
Sum 92. 08 = $6^{h}\ 8\frac{1}{2}$

Which deducted from $6^{h}.\ 41'$, leaves $32'\frac{1}{2}$ for the Difference of the Angle at the Pole in Time. Say then $46\llcorner 4 : 3\llcorner 64 :: 32'\llcorner 5 : 25'\llcorner 4$ which is the Difference in time of the Meridians of *Greenwich* and *Dublin*.

For *Paris* thus:

| | ° | ′ | | | |
|---|---|---|---|---|---|
| $VC=$ | 58 | 22 | R. | 10. | |
| $VP=$ | 27 | 00 | S. | 9.657047 | |
| $PC=$ | 41 | 48 | S. | 9.823821 | |
| Sum of 3 | 127 | 10 | | 19.480868 | |
| Half Sum | 63 | 35 | | 9.952105 | |
| Diff. of $VC$ | 5 | 13 | | 8.958670 | |
| Double Radius | | | | 20. | |
| Sum | | | | 38.910775 | |
| Remainder | | | | 19.429907 | |
| Half Remainder | | | | 9.714953 | $=$ C S. 58°. 45′ |

double 117 30
As before, Remainder 62 30
Add, Primary Angle 40 44
Sum 103 11
In Time $6^h\ 52\frac{1}{2}$

From which deduct 6°. 41′. the Remainder $11'\frac{1}{2}$ is the Difference of the Angle at the Pole in Time. Say then, $4_L64 : 3_L64 :: 11\frac{1}{2} : 9'$. which is the Difference in Time, of the Meridians of *Greenwich* and *Paris*.

*Corol.* (1.) The latter Branch of the Problem determines the Hour and Minute when the Centre of the Eclipſe croſſes the Meridian at any aſſign'd *Latitude*; and by a very ſmall Allowance when the very middle of the Eclipſe, or of Total Darkneſs happens in any Place very near the ſame.

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*Corol.*

*Corollary* (2.) The ſame Branch determines the true *Difference of Meridians* in all Places over or near which the Center of the Eclipſe paſſes, and ſo the Diſtance, *Eaſt* and *Weſt* from any known Meridian whatſoever. Thus becauſe the Meridians of *Paris*, *Greenwich*, and *Dublin* do hence appear to be different as to the Angles of the Pole from the Primary Meridian, in the foregoing Angles, 62°. 28'. 59°. 31'. and 51°. 24'. reſpectively; It is plain, that the reſpective *Longitude* of *Paris* and *Greenwich*, when reduc'd from Angles of the Pole to the Difference of Meridians, is 2° 19'. and that of *Greenwich* and *Dublin* 6°. 22'. and by Conſequence of *Paris* and *Dublin* 8°. 41'. Which is no other than the Foundation of my grand *Problem* of *Diſcovering the Geographical* Longitude *of Places by Solar Eclipſes, from the* Latitude *given*: Of which more hereafter.

## XV.

To find the Duration of a Solar Eclipſe, along or near the Path of the Moon's Center, in any Place whatſoever.

From the Motion of the Moon from the Sun gain the Duration of the entire Eclipſe; or the Time of that Center's Paſſage over the Diameter of the Penumbra, if there were no diurnal Motion during that Time, thus:

As the hotary Motion of the Moon from the Sun, which is in angular Meaſure 35°. 18'. and is given in the Calculation; to an Hour or 60' in Time :: So let the Diameter of the Penumbra there given, alſo in angular Meaſure = 65°. 10'. be to a fourth Number: which will be the Number of Minutes requir'd. In Decimals thus:

$$35'_{\llcorner}3 : 60' :: 65'_{\llcorner}17 : 110'_{\llcorner}8 = 1^{h} : 50\tfrac{4}{5}.$$

But then, becauſe the diurnal Motion of the Earth muſt be compounded with this rectilinear Motion of the Moon; and that as we have already ſtated it, this Eclipſe will croſs the Meridian of *Greenwich* 41′ paſt 6 a-Clock in the Evening; the one Part of the Duration of the Eclipſe being before, and the other Part after that Time, both Intervals muſt be unequally affected by the diurnal Motion, and we muſt then take the former half-Duration 55′. 24″ diſtinctly. And ſince the diurnal Motion of *Greenwich* is in a larger Parallel than that of the Center of the general Eclipſe; while its Obliquity to the Path of the Shadow's Center increaſes; their reſpective Motions will nearly keep the ſame Proportion all along; and ſo we may ſafely omit the Conſideration of them both. We have alſo already diſcovered, that the Velocity of this Center's Motion is here, to that of the Velocity of the diurnal Motion, As 829⌞6 to 178⌞4. or as 4⌞64 to 1. And becauſe the Eclipſe begins about 14′ before 6, 14′ after 6 ballances the ſame; and theſe 28 Minutes are almoſt all one, as if there were no diurnal Motion at all. So that we have only 27′ 24″ capable of Retardation in the firſt half: The middle Time of which will be about 30′ after 6. We muſt therefore look into the Table, *p.* 22,——26 for the *Arc* 97½, or its Supplement 82½, correſponding to 89¼ of the Sines. Where the Difference of half a Degree is inſtead of 19⌞1, as about Noon, no leſs than 160, which multiplied by 4⌞64 comes to 7424. So that the Motion is here retarded a 39th Part. Say then, As 39 to 38, ſo is 27⌞4 to 26⌞7 which 26⌞7 or 26′ 42″ is to be added to the 28 before excepted, for the Duration of the former Part of the Eclipſe = 54′ 42″. The Middle of the ſecond Part will be about 73 = 18°¼ backward beyond 90°. that is, 71° ¾ of the Arc, which correſponds to 85° ½ of the Sines, where the Difference of half a Degree is 61° ½. This multiplied by 4˪64 gives 285. So that the Retardation is as 19˪1 to 285, or 1 to 14˪9. Say then, As 14˪9 to 13˪9 : ſo is 55˪4 to 51˪7 = 51′ 42″, which is the Duration of the latter Part of the Eclipſe, and added to the former Part of the Duration, gives us the whole Duration ≐ 106′ 24″ = $1^{h}$46′ 24″, without the Conſideration of the Elevation of the Luminaries above the Horizon, which a ſmall Matter enlarges that Duration : Of which hereafter.

*N. B.* If we would be ſtill more preciſely nice, we may diſtinctly allow for the Difference of the Parallels, and the different Obliquity of the Direction; of which *p* 26, 27. before : Which yet are here omitted, as very inconſiderable.

## XVI.

To find the Duration of the Total Darkneſs along the Path of the Moon's Center; if the Luminaries were in the Horizon.

Say, As the Moon's Motion from the Sun in an Hour, or 60′; in the Calculation = 35′ 18″ is to thoſe 60′ : So is the Difference of the Diameters of the Sun and Moon in the ſame Calculation, = 1′ 38″ to a fourth Number : Which will be the entire Duration of the Total Darkneſs if the Luminaries were not at all elevated above the Horizon. In Decimals thus :

35′˪3 : 60′ :: 1′˪62 : 2′˪73 = 2′46″⅔.

## XVII.

To find the Altitude of the Sun above the Horizon, when the Center of the Shadow croſſes the Meridian at *Greenwich*.

In the Triangle (*Fig.* 10.) *Z P S*, *Z P* is = Diſtance between the Zenith and Pole of the Earth : 38° 30′. *P S* is = Diſtance between the Pole of the Earth and Center of the Sun : = Complement of the Sun's Declination, or to 69° 28′. And the Angle *Z P S* = interval of Time, from the Meridian = 6ʰ 41′ = 100° 15′, whoſe Supplement is 79° 45′.

Say then,

|  | ° | ′ |  |
|---|---|---|---|
| R. |  |  | 10. |
| CS. *ZPS* | 79 | 45 | 9. 250280 |
| T. *ZP* | 38 | 30 | 9. 900605 |

T. P *R* = 19. 150885 = 8° 3′ = P *R*

+ 69 28 = P *S*

= 77 31 = *RS*

And,

|  | ° | ′ |  |
|---|---|---|---|
| CS. *P R* | 8 | 3 | 9. 995699 |
| CS. *Z* P | 38 | 30 | 9. 893544 |
| CS *R S* | 77 | 31 | 9. 334766 |
|  |  |  | 19. 228310 |

CS. *Z S* = 9. 232611 = 80° 10′ = *Z S*

*Ergo*, Altitude = 9 50

## XVIII.

To find the Azimuth of the Sun at the ſame Time and Place.

In the former Triangle the Complement of the Altitude being now found = *Z S* = 80° 10′. Proceed thus:

|  | ° | ′ |  |
|---|---|---|---|
| S. *PS* | 80 | 10 | 9. 993572 |
| S. *Z P S* | 79 | 45 | 9. 993013 |
| S. *P S* | 69 | 28 | 9. 971493 |
|  |  |  | 19. 964506 |

S. P*ZS*=Suppl. = 9. 970934 = 69° 16′ = P*Z*

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XIX.

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## XIX.

To find the Number of Minutes that any Solar Eclipſe extends to, in a plain Perpendicular to the Axis of the Penumbra.

When the Parallax of the Moon is 57′ 17″, which is near its mean Quantity; the Moon's Diſtance is 60 Semidiameters of the Earth: And one Second on the Diſk of the Earth as viewed at the Diſtance of the Moon; or on the Diſk of the Moon viewed at the Diſtance of the Earth, is exactly one geographical Mile. And 60″ or one Minute, is one Degree upon the Diſk of the Earth or Moon. But when the Moon's Parallax, as in the Calculation of this Eclipſe, is 61′. 48″ whoſe Sine is $\frac{17978}{1000000}$ or $\frac{1}{55,63}$ of the Radius nearly: One Second is leſs than a Mile; and that in the Proportion of 55,5 to 60. So that about 65 geographical Miles correſpond to 60″, or one Minute: And accordingly, 1781 ſuch Miles will correſpond to 32′ 35″, which in that Calculation, is the Semidiameter of the Penumbra, or the utmoſt perpendicular Extent of the Eclipſe.

*N. B.* We may always make uſe of the foregoing Tables, *pag.* 29, —— 34 for the perpendicular Diſtances from the Path of the Moon's Centre, at all Durations of the Eclipſes, and of Total Darkneſs, by ſtill calling the Radius of every *Penumbra* 2000 equal Parts or Miles, and the Radius of every *Umbra* or Total Shadow, 50 ſuch Parts or Miles; and uſing them accordingly. And this without any other Inconvenience than the Suppoſition of other than geographical Miles: Which Inconvenience, by the Reduction of them to geographical Miles afterwards, will always, as here, come to nothing.

XX.

## XX.

To find the Alteration there is in the Extent and Duration of Eclipſes, and of total Darkneſs, on Account of the Elevation of the Luminaries at that Time above the Horizon.

This is ever, as the Radius: to the Sine of the Sun's Altitude: As compared with the particular Diſtance of the Moon at that Time. Thus at the Middle of this Eclipſe at *Greenwich*, the Altitude of the Sun has been found to be 9°. 50′, whoſe Sine is $\frac{1708}{10000}$, which divided by 55′,5 is equal to $\frac{1}{3}$ of the whole: And in the Semidiameter of the *Penumbra*, as well as *Umbra*, amounts to about 6 Miles every Way. Thus alſo at the Middle of the Eclipſe in *North-America*, where 'tis Central at Noon, the Sun will be about 52° above the Horizon: whoſe Sine is $\frac{788}{1000}$, which divided by 55,5 is $\frac{1}{70}$ nearly; which in the ſame Semidiameters, amounts to about 29 Miles every way. And for the Duration at *Greenwich* add to the common Duration already found, or to $1^{h}$ 46′ 24″ the $\frac{1}{324}$ Part of the ſame, or about $\frac{1}{3}$ of a Minute both ways: which is about $\frac{2}{3}$ in the whole; which will bring the entire Duration from $1^{h}$ 46′ 24″ to full $1^{h}$ 47′; and for Total Darkneſs will add about $\frac{6}{50}$ of the whole total Darkneſs = 20″, and increaſe the Duration from 2′ 46$\frac{2}{3}$ to 3′ 6″ $\frac{2}{3}$. But for the Alteration in *North America*, which is about $\frac{1}{70}$ both ways, or $\frac{1}{35}$ in the whole; this will there increaſe the entire Duration 4′ 12″, and from $1^{h}$ 46′ 24″ bring it to $1^{h}$ 50′ 36″, and for the total Darkneſs will add about $\frac{22}{50}$ of the whole, or bring the 2′ 46″ $\frac{2}{3}$ to 4′ 47″.

## XXI.

## To find the Species and Dimenſions of the Ellipſis made by the Total Shadow at the ſame Places and Altitudes.

The ſhorter Axis of the Ellipſis, (which is the ſame with the Diameter of the Conick Shadow it ſelf, in a Plane Perpendicular to the Axis ) is to the Longer; as the Sines before mentioned. Or at *Greenwich*, as 1708 to 10000; and at *North America*, as 788 to 1000. And ſince that ſhorter Axis is in the Calculation 98 Minutes or Miles, of 65 to a Degree, they muſt be equal to 90 Geographical Miles; which by the laſt Problem muſt be increas'd to 96 and to 119 reſpectively. The golden Rule will therefore ſoon ſhew the Length of the longer Axis of the Shadow in both Places.

For *Greenwich* 1708 : 10000 :: 96 : 562.
For *North-America* 788 : 1000 :: 119 : 151.

## XXII.

## To determine the central Point in the Elliptick Shadow for *Greenwich*, at the Middle of the ſame Eclipſe.

By *Lemma* XIX. and in its Figure, As *VB*: to *VA* :: So is the Sine of *BAV* = 9° 34′ = 166 : to the Sine of *ABV* = 10° 6′ = 175, and ſo is *BC* : to *AC*. And *componendo*, As 166 + 175 = 341 to 175 : So is *BC* + *CA* = 562 to *CA*, which therefore is equal to 289. Alſo As 166 + 175 = 341 to 166 : So is *BC* + *CA* = 562 to *BC*; which therefore is equal to 274.

For 341 : 175 :: 562 : 289.
And 341 : 166 :: 562 : 274.

Whence the Parts of the longer Axis being ſound, the larger $CA = 289$ Miles: the leſſer $BC = 274$ Miles. The half of which, $DC$ is the Diſtance between the central Point of the Shadow $D$, and the Center of the Ellipſis $C$, $= 7\frac{1}{2}$ Miles.

## XXIII.

**To determine the Angle of Direction of the Total Shadow over the Meridian of *Greenwich*.**

This is to be done either by Conſtruction, upon drawing the Curve Line of this Motion, through the ſeveral Points where, by Calculation, the Center of the Shadow croſſes the Meridians of *Paris*, *Greenwich*, and *Dublin*; and meaſuring the Angle it makes with the ſeveral Meridians; or more exactly for *Greenwich* by ſolving the Triangle (*Fig.* 11.) $PVO$, where the Arc $VO$ is equally diſtant *Weſt*, as *Paris* is *Eaſt* from *Greenwich*, is to be found by the Method already made uſe of; thus:

*Paris* Angle at the Pole $= 62°\ 28'$. Angle for $OV$ at the Pole is leſs by twice the Difference of the Angles at the Pole for *Paris* and *Greenwich* $= 5°\ 54'$, And $62°\ 28' - 5°\ 54' = 56°\ 34'$. Firſt then find the Arc $VO$ = Compl. Latitude for O, as in *Probl.* XIV. by this Analogy.

| | | | | |
|---|---|---|---|---|
| R. | | | 10. | |
| | ° | ' | | |
| CS. $QPC$ | 56. | 34 | 9.741129 | |
| T. $PV$. | 27. | 00. | 9.707166 | |
| T. $PR$. | | | 9.448291 | $= 15\ 41$. |

Then ſay,

° '
CS. *PV.* 27. 00. 9. 949880
CS. *PR.* 15. 41. 9. 983523
CS. *VG.* 58. 22. 9. 719730
19. 703253 ° '
CS. *PC* 9. 753373 = 45. 29.
Deduct 15. 14.
Rem^t. *VO.* 39. 48.
*VG.* 40. 46.
*VP.* 41. 48.

Then in the Triangle *VOP* whoſe included Angle is 4° 38′, and whoſe two Sides are 39° 48′ and 41° 48′. Find the Arc *PR* as uſual thus;

R. 10.
° '
CS. *OVR.* 04. 38. 9. 998577
T. *VO.* 39. 48. 9. 920733 ° '
T. *VR.* 19. 919310 = 39 48
from 41 42
Remn^t 2° 6′ = *PR.*

Then for the Angle *VPO*, ſay:

° '
S. *PR.* 02. 06. 8. 563999
S. *VR.* 39. 42. 9. 805343
T. *OVR.* 04. 38. 8. 908719
18. 714062 ° '
T. *VPO.* = 10. 150063 = 54. 42.

And for *VOR*;

| | ° | ′ | | | | |
|---|---|---|---|---|---|---|
| S. *VO*. | 39. | 42. | 9. 805343 | | | |
| S. *VPO*. | 54. | 42. | 9. 911763 | | | |
| S. *VP*. | 41. | 48. | 9. 823821 | | | |
| | | | 19. 735584 | | | |
| S. *VOP* = | | | 9. 930241 | = | 58. | 23 |
| | | | | + | 113 | 05 |
| | | | | Half | 56 | 32½ |
| | | | | Compl. | 33 | 27½ |

So that the Acute Angle *VGO*, which the Courſe of the Center of the Shadow makes with the Meridian of *Greenwich*, is 56° 32′ ½; And by conſequence the Angle it makes with the Parallel of Latitude there is 33° 27′ ½, 5 or 6 Degrees more than in Dr. *Halley*'s Scheme.

*Corollary*. The longer Axis of the total Shadow at the Meridian of *Greenwich*, makes an Angle of 12° 43′ ½ with the Direction of the Shadow.

| | | |
|---|---|---|
| For if from the Angle that the Direction of the Center of the Shadow makes with the Parallel of *Greenwich*, | 33 | 27½ |
| We deduct the Angle that the longer Axis makes with the ſame Parallel, which is the Azimuth formerly diſcovered, | 20 | 44 |
| There remains this Angle | 12 | 43½ |

## XXIV.

To Determine the perpendicular Breadth of the Total Shadow, when that Shadow is neareſt to *London*; or when the central Point is about the Meridian of *Padſtow* in *Cornwall*.

This *Problem* is, In an Ellipſis of 96 Miles broad, and 562 Miles long; whoſe longer Axis makes an Angle with the Direction of the Shadow of 12° 43 $\frac{1}{2}$; To determine the Length of the greateſt Perpendicular that falls on that Diameter; along which the Direction of the Motion is. For twice that Perpendicular is the Breadth required. Now I have actually drawn this Ellipſis, or total Shadow, for my Map of this Eclipſe. And I find by that Conſtruction, that the Breadth of the total Shadow over *England* is about 155 Miles; or 30 Miles broader than in Dr. *Halley*'s Scheme. Time will probably diſcover which is neareſt the Truth.

*N. B.* This total Shadow increaſes ſo greatly in Length as it goes *Eaſtward*, that it will reach from *Paris* to the utmoſt Boundary of the Sun-ſetting at once. And though the Center of the Shadow, by my Calculation, will end 8° $\frac{1}{3}$ *Eaſt* of the Meridian of *London*, in the Latitude of 45° 9′, or in the *Alps*, not far from *Milan* and *Turin*; Yet by reaſon of the Extenſion and Breadth of the total Shadow, and the Refraction of the Rays of Light near the Horizon, it ſeems probable to me, that all *Switzerland* and *Lombardy*, as far as *Trent*, *Mantua*, *Cremona*, and *Parma*; nay, perhaps, as far as *Venice*, *Padua*, *Bononia*, *Ferrara*, *Ravenna* and *Florence*; may at the ſame Time be invelop'd in the Total Shadow; and that the Sun may ſet eclipſed at all or moſt of thoſe Places at once.

## XXV.

To determine the Digits eclipſed, and the Duration of the Eclipſe, or of total Darkneſs, at any given Diſtance from the Path of the Moon's Center; and *vice verſa*. Theſe Digits and Durations, if we do not conſider the diurnal Motion, are immediately found to any given Diſtance in the Tables, P. 29, —— 34. Thus, if the Digits be 4, or Duration 80′ 24″, the firſt Table gives us the Diſtance of 1350 Miles. And, *vice verſa* if the Diſtance be known to be 1350 Miles, the ſame Table gives us 4 Digits and 80′ 24″ Duration. And the like is true of the total Shadow, and the Diſtance or Duration in the ſecond Table. Thus alſo we learn from the Map of this Eclipſe, that the neareſt Approach of the total Shadow to *London* will be about 40 geographical Miles: Which 40 Miles, in this Obliquity of the Motion, is about 31 perpendicular Miles. So that [illegible] = [illegible] of a Digit, is the Quantity of the Rim of Light that will be ſeen at *London*, when the Eclipſe is greateſt: And by conſequence the Digits there eclipſed will be 11$\frac{48}{100}$. Nor does it appear to me, that the total Darkneſs will come nearer to *London*, than 40 Miles; although Dr. *Halley*'s Scheme brings it within 7 or 8 Miles. As for the Duration of the entire Eclipſe here, it will be much the ſame as if it were Central: And has been already determined to 1ʰ 47′. And for the total Darkneſs along the Path of the Center between *Exeter* and *Plimouth*, its Duration will be, as before ſtated, 3′ 6″ ½.

*N. B.* As to the Alterations which will ariſe in more conſiderable Diſtances from the Path of the Moon's Center, proceed thus: Firſt, find the Minutes of Duration correſponding to any given Diſtance in Miles, *e. g.* 1350, thus; 2000 : 1350 :: 32′,63 : 22′,02. Or, As 2000 Miles, the Radius : to 1350 the Sine :: So are the Number of Mi-

Minutes for the Semidiameter of the Penumbra: to the Number of Minutes for that Sine. Now $32' 19''$ the preſent Length of the greateſt Diſtance $- 22' 01'' = 10' 18''$. Then compute every Thing as if the Center of the Shadow were $10' 18''$ Latitude or neareſt Diſtance; and as if the Chord of that Arc over the Earth's Diſk were the Path of the Moon's Center; and the Chord of the Diſtance from the Center of the Penumbra were the Diameter of the Penumbra: And all will be diſcovered by the Rules before going; without any new Directions whatſoever. Only if this Diſtance be the contrary way to the preſent Example, we muſt here make uſe of Addition inſtead of Subſtraction.

*N. B.* Upon conſulting the *French* Ephemerides of *Des Places*, recommended, in ſome Degree, by *Caſſini* himſelf, I find that he determines the Eclipſe at *Paris* thus.

| | h | ′ | ″ |
|---|---|---|---|
| Beginning, *May* 22. (*N. S.*) | 06 | 03 | 00 |
| Middle | 06 | 58 | 38 |
| End | 07 | 54 | 00 |
| Whence the Duration there is | 01 | 51 | 00 |
| Digits eclipſs'd | 10 $\frac{17}{60}$ | | |
| Latitude of the Moon at the Middle | $39' \frac{1}{2}$ | | |
| Difference from me in Digits | .01 $\frac{31}{60}$ | | |
| In Latitude | 07 10 | | |

If theſe Digits and this Latitude be right, Dr. *Halley*, and I, and thoſe *Engliſh* Aſtronomers that have computed and conſtructed this Eclipſe, who all, I think, do agree, that it will be total at *Paris*, and that for two or three Minutes alſo, are prodigiouſly miſtaken. For if *Des Places* be in the right, *Copenhagen* and *Stockholm* will ſtand much fairer for the Pretence to a total Eclipſe, than either *Dublin*, *London*, or *Paris*. Time will certainly determine who are in the right.

*N. B.* If we add the Duration of the Eclipſe, conſider'd without Regard to the diurnal Motion, to the Time of the Center of the Shadows paſſing over the entire Diſk, we gain the entire Duration of the Eclipſe in general, thus;

| | | | |
|---|---|---|---|
| To | 1 | 50 | 48 |
| Add | 2 | 58 | 24 |
| Sum | 4 | 49 | 12 |

Therefore the general Eclipſe by the Meridian of *London*,

| | h | ' | " |
|---|---|---|---|
| Begins | II. | 52 | 24 |
| Middle | V. | 17 | 00 |
| End | VII. | 41 | 36 |
| Duration | 4h | 49 | 12 |

Eclipſe at *London*,

| | | | |
|---|---|---|---|
| Begins | V. | 45 | 00 |
| Middle | VI. | 41 | 00 |
| End | VII. | $32\frac{1}{2}$ | 00 |
| Duration | 1h | $47\frac{1}{2}$ | 00 |
| Digits eclipſed | 11 | $\frac{48}{60}$ | |

| | | |
|---|---|---|
| Duration of Total Darkneſs between *Exeter* and *Plimouth*, | 03 | $6\frac{1}{2}$ |

*N. B.* Dr. *Halley*'s Times are ſtill about 5' ſooner than mine, and his general Duration about 7', and his Duration at *London* about 1" longer.

*N. B.* I cannot tell the Reaſon why my Original Calculation of this general Eclipſe, which has been carefully made according to Sir *Iſaac Newton*'s famous *Theory of the Moon*, does here ſo much differ from Dr. *Halley*'s Determination, as $5'$ in Time; eſpecially ſince both thoſe Methods did very well agree, in the laſt celebrated total Eclipſe, *Apr.* 22. 1715. 'Tis Time alone that can determine between theſe two Methods of Calculation.

*N. B.* I have lately been ſhewed an exact Scheme of this next Eclipſe, according to Mr. *Flamſteed*'s own Tables and Determination, and made in his Life-time; wherein the Digits eclipſed are $11\frac{8}{60}$ exactly according to my Determination in this Paper.

# A PROPOSAL

## For the Diſcovery of the LONGITUDE of the ſeveral Places of the Earth, by Total Eclipſes of the Sun.

IT is humbly propoſed, That Obſervations be made in all Places where Solar Eclipſes are ſeen, of the exact *Duration* of the ſame; by either viewing the Beginning and Ending thereof through a Teleſcope, with a Glaſs ſmoaked in the Flame of a Candle, for ſaving the Eye of the Obſerver; or elſe by caſting the Sun's Image through ſuch a Teleſcope upon white Paper, and viewing the firſt and laſt Impreſſion of the Moon's Shadow upon it. And that the Hour, Minute, and Second of ſome Pendulum Clock be carefully noted at the ſame Time: And that when the ſame Obſervations are tranſmitted for the Uſes of Geography, the Latitudes of the Places be alſo ſet down and tranſmitted at the ſame Time: That the like Obſervations be alſo made in all Places where ſuch Eclipſes are total and viſible, of the exact Beginning and Ending, with the Duration of Total Darkneſs; by the like Compariſon of a Pendulum Clock, or other pendulous Body that vibrates Seconds, or half Seconds; and, with the Latitude be tranſmitted in the ſame Manner, and for the ſame Uſes. How by the Help of theſe Ob-

*A Table of the* Latitude *of the Center of the total Shadow at the Eclipſe,* May 11. 1724. *to every* 10 *Degrees of the Angle about the Pole, and every* 7°˪845. Longitude *from* London.

| Angles at the Pole. | | | Longitude from *London.* | | Latitde. | |
|---|---|---|---|---|---|---|
| ° | ' | | ° *Eaſt.* | | ° | ' |
| 70 | 19 | | 8˪577 | | 45 | 9 |
| 69 | 23 | | 7˪845 | | 45 | 34 |
| | | | Weſt. | | | |
| 59 | 23 | | 0000 | | 49 | 17 |
| 49 | 23 | | 7˪845 | | 52 | 19 |
| 39 | 23 | | 15˪690 | | 54 | 42 |
| 29 | 23 | | 23˪535 | | 56 | 29 |
| 19 | 23 | | 31˪380 | | 57 | 43 |
| 09 | 23 | | 39˪225 | | 58 | 13 |
| 00 | 00 | | 46˪600 | | 58 | 38 |
| 00 | 37 | | 47˪070 | | 58 | 38 |
| 10 | 37 | | 54˪915 | | 58 | 21 |
| 20 | 37 | | 62˪760 | | 57 | 36 |
| 30 | 37 | | 70˪605 | | 56 | 18 |
| 40 | 37 | | 78˪450 | | 54 | 27 |
| 50 | 37 | | 86˪295 | | 51 | 59 |
| 56 | 22 | Midle. | 90˪021 | | 50 | 16 |
| 60 | 37 | | 94˪140 | | 48 | 51 |
| 70 | 37 | | 101˪985 | | 45 | 5 |
| 80 | 37 | | 109˪830 | | 40 | 40 |
| 90 | 37 | | 117˪675 | | 35 | 56 |
| 100 | 37 | | 125˪520 | | 30 | 31 |
| 110 | 37 | | 133˪365 | | 25 | 15 |
| 120 | 37 | | 141˪210 | | 20 | 11 |
| 130 | 37 | | 149˪055 | | 15 | 37 |
| 136 | 59 | | 154˪052 | | 13 | 3 |

*N. B.* Here, as well as in the next Table, *The Angles at the Pole* differ by Ten Degrees; and the *Longitude from London* is found by the Proportion of 4˪64 to 3˪64 to be 7°˪845 of Longitude from *London* for every ſuch Ten Degrees. But if, as is generally moſt convenient, we would have the Differences of *Longitude from London* be even 10 Degrees; We muſt find the correſpondent *Angles at the Pole* before we begin our Calculations by the inverſe Proportion of 3,64 : 4,64 :: 10°: 12'¼ the perpetual Addition of which Number will give us a Series for ſuch a Calculation.

*A Table of the Latitude of the Center of the total Shadow at the Eclipſe,* Sept. 4. 1717. *to every* 10 *Degrees of the Angle at the Pole; and every* 7°,167 *Longitude* from *London.*

| Angles at the Pole. | | | Longitude from *London.* | | Latitde. | |
|---|---|---|---|---|---|---|
| ° | ' | | ° *Weſt.* | | ° | *North* |
| 11 | 33 | | 12,248 | | 30 | 18 |
| 4 | 28 | | 7,167 | | 30 | 20 |
| | | | Eaſt. | | | |
| 00 | 00 | | 3,963 | | 30 | 14 |
| 5 | 32 | | 0,000 | | 29 | 28 |
| 15 | 32 | | 7,167 | | 27 | 56 |
| 25 | 32 | | 14,334 | | 25 | 40 |
| 35 | 32 | | 21,501 | | 22 | 38 |
| 45 | 32 | | 28,668 | | 18 | 27 |
| 55 | 32 | | 35,835 | | 14 | 24 |
| 65 | 32 | | 43,002 | | 9 | 21 |
| 75 | 32 | | 50,169 | | 5 | 4 |
| 77 | 58 | Midle. | 51,903 | Midle. | 3 | 57 |
| 85 | 32 | | 57,336 | | 0 | 16 |
| | | | | | South. | |
| 95 | 32 | | 64,503 | | 4 | 32 |
| 105 | 32 | | 71,670 | | 9 | 55 |
| 115 | 32 | | 78,837 | | 14 | 54 |
| 125 | 32 | | 86,004 | | 19 | 16 |
| 135 | 32 | | 193,171 | | 23 | 00 |
| 145 | 32 | | 100,338 | | 25 | 57 |
| 155 | 32 | | 107,505 | | 28 | 8 |
| 165 | 32 | | 114,672 | | 29 | 34 |
| 168 | 39 | | 116,909 | | 30 | 14 |

3,53 : 2,53 :: 10 : 7,167.
2,53 : 3,53 :: 10 : 13,952.

*A Cal-*

## *A Calculation of the Total Eclipſe of the Sun,* A. D. 1724. May 11. *Poſt Meridiem.*

| | | M.'s mean Mo. | Moti$^{n}$ of Apogee. | Mot$^{n}$ of N$^{odes}$ r tr. |
|---|---|---|---|---|
| | | s ° ′ ″ | s ° ′ ″ | s ° ′ ″ |
| *Anno Dom.* | 1701 | 10 15 20 00 | 11 8 20 00 | 4 27 24 20 |
| Years | 20 | 4 13 34 05 | 3 3 50 15 | 26 50 15 |
| and | 3 | 28 9 10 | 4 1 59 32 | 1 27 59 9 |
| Leap-Y. *May,* Days | 11 | 9 29 17 03 | 14 42 21 | 6 59 24 |
| Hours | 5 | 2 44 42 | 1 24 | 40 |
| Minutes | 15 | 8 14 | 4 | 2 |
| (Mean Time.) | | | | 3 1 49 30 |
| Moon's mean Motion | | 1 29 13 14 | 6 28 53 36 | 1 25 34 50 |
| Sun's mean Anomal. | | 10 22 19 57 | An.Eq.add 12 2 | An.Eq.ſubſt. 5 43 |
| Phyſic. Parts ſubſtract | | 7 7 } | 6 29 5 38 | 2 25 29 7 |
| 2d Equat. ſubſtract | | 2 31 } 9 38 | Mean Pla. correct | Mean Pla. correct |
| 3d Equat. add | | 10 } | 2 1 38 45 | 2 1 38 45 |
| 6th Equat. add | | 2 6 } 2 16 | Sun's true Place | Sun's true Place |
| Diff. ſubſtr. | | 7 22 | 7 2 33 7 | 6 9 38 |
| Moon's mean Pl correct | | 1 29 5 52 | Annual Argument | S's diſt. from Node |
| True Pl. of Apog. ſubſtr. | | 7 9 14 38 | 10 9 0 | Equat. add 18 56 |
| Moon's mean Anomal. | | 6 19 51 14 | Equat. add | 1 25 48 3 |
| Equat. add | | 2 31 7 | 7 9 14 38 | True Pl. of Node |
| Moon's true Pl. in Orbit | | 2 1 36 59 | True Pl. of Apogee | 5 17 9 |
| Sun's true Pla. ſubſtract | | 2 1 38 45 | Eccentricity 6000 | Inclinat. Limit. |
| Moon's diſt. from Sun | | 11 29 58 14 | ′ ″ | ′ ″ |
| Nodes true Pl. ſubſtract | | 1 25 48 3 | M's h. mot. 37 42 | S mid. Sun 15 53 |
| Argument of Latitude | | 5 48 56 | Sun's 2 24 | Sem Moon 16 42 |
| Reduction ſubſtract | | 1 30 | M's fro. Sun 35 18 | Sem. Pen$^{bra}$ 32 35 |
| Moon's tr. Pl. in Eclip. | | 2 1 35 29 | 35′ 18″ : 60′ :: 1′ | Semi. Diſc. 61. 38 |
| Moon's true *North* Lat. | | 32 14 | 46″ : 3′ 2″ | Angle of the M's |
| At the Eclipſe | | 32 19 | Eq. time add 3 53 | Way with the |
| | | | Reduct. ſubſt. 2 33 | Eclіptick 5 36 |
| | | | Diff. add 4 22 | Diff. Diameter of |
| | | | *Ergo,* Eclipſe is | the Sun & Moon, |
| | | | 11$^{d}$ 5$^{h}$ 19′ 22″ | or Breadth of |
| | | | Temp. appar. | total Shad. 0 98 |
| | | | deduct the Error | |
| | | | of the Tables a- | |
| | | | bout 2′ 22″ | |
| | | | true Tim. 5$^{h}$ 17 0 | |

## *A Calculation of the Sun's Place for the ſame Time.*

| | Sun's mean Motion. | Motion of Perihelion. |
|---|---|---|
| | s ° ′ ″ | s ° ′ ″ |
| *A. D.* 1701 | 9 20 43 40 | 3 7 44 10 |
| Years 20 | 9 4 | 21 0 |
| And 3 | 11 29 17 0 | 3 9 |
| Leap-Y. *May,* Days 11 | 4 10 6 19 | 23 |
| Hours 5 | 12 19 | |
| Minutes 15 | 37 | 3 8 9 2 |
| (meanTime) | | Place of Perihelion. |
| Sun's mean Motion | 2 0 28 59 | 2 90 28 59 |
| Equation add | 1 9 46 | Moon's mean Motion. |
| Sun's true Place | 2 1 38 45 | 10 22 19 57 |
| | | Sun's mean Anom. |

## *A Calculation of the Total Eclipſe of the Sun*, A. D. 1727. Sept. 4. Mane.

| | | M's mean Mot. | | | | Motion Apogee. | | | | Mot$^{n}$ of N$^{ode}$ retr. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | s | ° | ' | " | s | ° | ' | " | s | ° | ' | " |
| *Anno Dom.* | 1701 | 10 | 15 | 20 | 0 | 11 | 8 | 20 | 0 | 4 | 27 | 24 | 20 |
| Years | 20 | 4 | 13 | 34 | 5 | 3 | 3 | 50 | 15 | | 26 | 50 | 15 |
| And | 6 | 2 | 9 | 28 | 55 | 8 | 4 | 5 | 44 | 3 | 26 | 1 | 29 |
| *Sept.* Days | 3 | | 1 | 23 | 35 | | 27 | 24 | 23 | | 13 | 1 | 37 |
| Hours | 20 | | 10 | 58 | 49 | | | 5 | 34 | | | 2 | 39 |
| Minutes | 35 | | | 19 | 13 | | | | 10 | | | | 5 |
| (Mean Time.) | | | | | | | | | | 5 | 5 | 56 | 5 |

| | | | |
|---|---|---|---|
| Moon's mean Motion | 5 21 4 37 | 11 13 46 6 | 11 21 28 15 |
| Sun's mean Anomaly | 2 15 32 16 | An.Eq.ſub. 18 59 | An. Eq. add 9 7 |
| Phyſical Parts add | 11 23 | 11 13. 27 7 | 11 21 37 22 |
| 2d Equat. ſubſtract | 0 38 | Mean Pl. Apo. cor. | MeanPl. of No. co |
| 6th Equat. add | 1 57 | 5 21 52 53 | 5 21 52 53 |
| Difference add | 12 42 | Sun's true Place | Sun's true Place |
| Moon's mean Pl. correct | 5 21 17 19 | 6 8 25 46 | 6 0 15 31 |
| True Pl. of Apog. ſubſt. | 11 16 23 44 | Annual Argum. | S.'s diſt. fro. Nod. |
| Moon's mean Anomaly | 6 4 53 35 | Eq$^{n}$.Ad 2 56 37 | Eq. Ad. 0 48 |
| Equation add | 42 26 | 11 16 23 44 | 11 21 38 10 |
| M.'s Correct. Pl. in Orb. | 5 21 59 45 | True Pl. of Apo. | True Pl. of Node |
| Sun's true Pla. ſubſtract | 5 21 52 53 | Eccentr. 66387 | 5 17 20 |
| Moon's Diſt. from Sun | 6 52 | | Inclin. Limit. |
| Variation add | 9 | S.'s hor. mot. 2 26 | |
| M.'s true Pl. in Orbit | 5 21 59 54 | Moon's 38 29 | Semid. S. 16 1 |
| Nodes tr. Pl. ſubſtract | 11 21 38 10 | M. fr. S. 36 2 | Semid. M. 16 50 |
| Argument of Latitude | 6 0 21 44 | 36,04 : 60 :: | Diff. = 00 49 |
| Reduction ſubſtract | 6 | 6,9 : 11' 30. | Sum = 32 52 |
| Moon's true Pl. in Eclip. | 5 21 51 48 | *Ergo*, Eclipſe will | Parallax M. Ho- |
| Moon's true *South* Latit. | 1 54 | be 20$^{h}$ 23 34' | rizon 62 7 |
| At the Eclipſe | 1 23 | Mean Time. | Sun 10 |
| | | Eqtim. ad. 4 38 | Semid. Diſk 61 57 |
| | | Reduct. ſubſtr. 9 | Angle of the M.'s |
| | | Diff. add 4 29 | viſible Way with |
| | | 20$^{h}$ 28 03 | the Ecliptick |
| | | Appar. Time. | 5 36 20 |

## *A Calculation of the Sun's Place for the ſame Time.*

| | | Sun's mean Motion. | | | | Motion of Perihel. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | s | ° | ' | " | s | ° | ' | " |
| *A. D.* | 1701 | 9 | 20 | 43 | 40 | 3 | 7 | 44 | 30 |
| Years | 20 | | | 9 | 4 | | | 21 | 0 |
| And | 6 | 11 | 21 | 33 | 8 | | | 6 | 18 |
| *Sept.* Days | 3 | 8 | 2 | 28 | 9 | | | | 40 |
| Hours | 20 | | | 49 | 17 | | | | |
| Minutes | 35 | | | 1 | 26 | 3 | 8 | 12 | 28 |
| ( Mean Time.) | | | | | | Place of Perihel. | | | |
| | | | | | | 5 | 23 | 44 | 44 |
| Sun's mean Motion | | 5 | 23 | 44 | 44 | Sun's mean Motion | | | |
| Equat. ſubſtract | | | 1 | 51 | 51 | 2 | 15 | 32 | 16 |
| Sun's true Place | | 5 | 21 | 52 | 53 | Sun's mean Anomaly. | | | |

Some

# Some Account of Obſervations lately made with Dipping-Needles, in Order to diſcover the LONGITUDE and LATITUDE at Sea.

UPON the Receipt of the liberal Aſſiſtance of His moſt Excellent Majeſty, King *GEORGE*; their Royal Highneſſes the Prince and Princeſs of *Wales*, and many other of the Nobility and Gentry, my kind Friends, I ſent laſt Year Four ſeveral Dipping-Needles to Sea; with Frames hung near the Center of Motion in Gimbols, to avoid the Shaking of the Ship; and with proper Inſtructions to the Maſters of the Veſſels: And this, in order to diſcover the State of Magnetiſm in the ſeveral Parts of the Globe; and to find whether accurate Obſervations could be made at Sea, and to determine whether the ſundamental Theory I laid down from former Obſervations would hold or not; *viz.* “That Magnetick Variation and Dip are all deriv'd from one Spherical Magnet in the Center of our Earth; with an irregular Alteration of the Variation, according to the different Degrees of Strength of the ſeveral Parts of the Loadſtone, as compounded with a very ſlow Revolution from *Eaſt* to *Weſt*: And with a regular Alteration of the Dip, nearly according to the Line of Sines, from the Magnetick Pole to the Magnetick Equator; the Axis of that Equator being ſufficiently Oblique

" to its Plane: All which is the Caſe of Sphe-
" rical Loadſtones here." Now having already received Four Journals from Four ſeveral Maſters employ'd, I take this Occaſion of returning my Benefactors hearty Thanks for their Aſſiſtance, and of giving them and the Publick ſome Account of the Succeſs of theſe Obſervations; and what Conſequences are naturally to be drawn from them; with the Difficulty hitherto met with in the Practice at Sea, and the proper Remedy for the ſame in future Trials.

Captain *James Jolly* ſet out in *July*, 1722. for *Archangel*, with one of my Dipping-Needles on Board. He, for ſome time, met with ſuch Difficulties in the Practice, as confin'd to the Frame I had given him, that he was not at firſt able to make any good Obſervations at all. But after ſome Time, he took the Needle into his own Cabin; and without any Approach to the Center of Motion, or any Contrivance for avoiding the Shaking of the Ship at all, having a clear and full Gale all along, but without any ſtormy Weather, He made me 28 very good Horizontal Obſervations, from the Latitude of 65 quite to *Archangel*: I ſay, *Horizontal Obſervations* only, as I deſired him; the Needle, by an Accident before he went, being rendred incapable of making any other with ſufficient Accuracy. In this Space the Needle altered its Velocity very greatly, as I expected it would: And 5 Vibrations which at firſt were perform'd in about 280″; beyond the *North-Cape* came to 250″; till towards *Archangel* it gradually returned to about 177″.

Captain *Othniel Beal* ſet out about the ſame Time for *Boſton* in *New-England*, with the ſame

Inſtrument, and made Four Obſervations of the Dip, both by the Vertical and Horizontal Vibrations, and by the Dip it ſelf; Three upon the open Sea, and One in the Haven of *Boſton*: Which in ſome ſmall Manner differed one from another, but in the main agreed, and kept the due Analogy I expected. He greatly complained of the Shaking of the Ship; till in *Boſton* Haven he made a nice Obſervation both Ways, which did not greatly differ: Tho' the greateſt Part of of his Obſervations by the Dip it ſelf were ſomewhat more agreeable to Analogy than the other. The Reaſon was, I take it, that, as he aſſured me, he always took great Care to avoid the Shaking of my Frame; which Frame tho' it very much avoided the ſlower and greater Oſcillation of the Ship, yet made a quicker but leſſer Oſcillation it ſelf: Which Fault I was ſufficiently ſenſible of juſt before the Ships were going away, but was not able then to obviate; as I am prepared to do hereafter. After Captain *Beal* had made and ſent me theſe Obſervations, he purſued his Voyage to *Barbados*, and thence to *Charles Town* in *South Carolina*; at both which Places he made Obſervations; but the beſt at *Barbados*. For before he came to *Carolina*, he obſerved the Axis of the Needle to ſhake; which made him take the Dip there otherwiſe than he ought to have done; which is the natural Occaſion that the Dip there did not ſo well agree to Analogy as the reſt. However, upon my Receipt of his firſt Journal, with the Four firſt Obſervations, eſpecially the exact one at *Boſton*, I formed a more exact Theory of the Proportion of the Alteration of the Dip in the Spherical Magnet of the Earth; and found it at this Diſtance of the Earth's Surface, not far from that in my Spherical Loadſtone, at the Diſtance of about $\frac{9}{10}$ of an Inch from its Surface; *viz.* Not exactly as the Line of Sines, where at the Middle of the Line the Angles are 60 and 30; but rather as 66 to 24. Which Rule therefore is what I now propoſe as much nearer than the other. By which Proportion I determined long before-hand the Dip at *Barbados* of 43°. or 44°, as many of my Friends can witneſs: And when Captain *Beal* delivered me the Paper of this Obſervation at *Barbados*, before I opened it, or in the leaſt knew what Dip it contain'd, I foretold to him from that Theory the very ſame Dip, which both himſelf and his Paper immediately aſſur'd me to be true; and whoſe Truth, as he inform'd me afterwards, was confirm'd by another Obſervation, made a little before in the open Sea, of about 45°.

Captain *Tempeſt* alſo, about the ſame Time, ſet out for *Antegoa* and *St. Chriſtopher's*, with the ſame Inſtrument and Frame. In his Letter, dated laſt *January*, he greatly complains of the Shaking of my Frame; and propoſes an Hint how it might be avoided: Which Method of its Avoidance I had long before thought of, and provided for accordingly; and which has been a full Year ready for Practice. Thoſe Obſervations of his, that I have yet received; for I have not heard from him ſince *January*, but hope ſoon to here farther; were but Three, and all at open Sea; and but one of them made both the Ways that I deſired: And, indeed, ſeem the leaſt agreeable to Analogy of any of the reſt. Only ſince that ſingle Obſervation, which was alſo made by the horizontal Vibrations and vertical Oſcillations, agrees very well to that Analogy; ſince they all three are about the ſame

Quantity of 8 or 9 Degrees exceed that Analogy; and ſince very near the ſame Place, where the third Obſervation was made, I have a double Obſervation of Captain *Beal*'s to correct the ſame; I rather conclude, that Captain *Tempeſt* made a Miſtake, and placed the wrong Edge of the Needle upward in all the Three Obſervations: Which would naturally occaſion ſuch a Difference. When I receive the reſt of his Obſervations, or his Needle again, I ſhall be able to judge better of that Matter. However, even theſe Obſervations agree in groſs with all the reſt, to the gradual Decreaſe of the Dip as you go nearer to the Equator: Tho' as they ſtand at preſent, they do not determine the accurate Proportion of that Alteration ſo well as the others.

Captain *Michel* alſo, long after the reſt, ſet out for *Hamburgh* with the ſame Inſtrument; though now without the Frame, which he was not willing to incumber himſelf with: and I ſuſpected that in its preſent Contrivance it did more hurt than help the Nicety of the Experiments. I alſo by him, ſent a Letter to the Reverend Mr. *Eberhard*, who was the Occaſion of my ſtudying this Matter, and was then Paſtor of *Altena*, cloſe by *Hamburgh*; deſiring that he would there make the Experiment very exactly, and give me a particular Account of it. But I have not yet received his Anſwer.

Now the Obſervations here mentioned, as well as thoſe many others I had by me before, do ſeem to me in general evidently to afford us the following Inferences:

(1.) That there is one Spherical Loadſtone, and but one in the Center of our Earth; and that this

Loadſtone, like other Spherical Loadſtones, has but one *Northern* Pole: Contrary to Dr. *Halley*'s Hypotheſis.

(2.) That this *Northern* Pole is ſituated, contrary to the ſame Hypotheſis alſo, a great Way to the *Eaſt* of our Meridian: And indeed, as I before had determined, about the Middle of the Diſtance between the *North Cape* and *Nova Zembla*. Captain *Jolly*'s numerous Obſervations prove this moſt fully: While in Sailing towards that Point his horizontal Vibrations greatly increas'd in Number: And when he turned almoſt at right Angles, as he went down to *Archangel*, they ſoon diminiſhed; and yet ſo little, after ſome time that it was evident he then ſailed not far from a Parallel to that *Northern* Pole; aud not very many Degrees from it neither; exactly according to my Expectations.

(3.) That the abſolute Power of the internal Magnet is conſiderably different in different Places; and that without any certain Rule; as it is upon the Surface of our *Terrella* or Spherical Loadſtones here. This the various Number of Seconds to a vertical Oſcillations, and all the Accounts in the other Obſervations fully prove; and by conſequence this muſt cauſe different Variations in different Places, as is the Caſe of our *Terrella*.

(4.) That there no where appears in open Seas any ſuch Irregularity in the Dip, as we ſometimes meet with near Shores, or at Land; and by conſequence that Dr. *Halley*'s grand Objection againſt the Diſcovery of the Longitude by the Dipping-Needle, taken from an Obſervation of his own, concerning ſuch an Irregularity near the Shore at *Cape Verd*; and from his own Hypotheſis of the four Magnetick Poles is utterly groundleſs.

leſs. Nor indeed ſhall I be at Reſt, till I have ſent a Dipping-Needle to *Hudſon's Bay*, on purpoſe to determine this Diſpute about the four Poles; For that Voyage being almoſt directly towards his ſecond *Northern* Pole all the way, and about the ſame Diſtance all the way from mine; if this Voyage afford much the ſame Dip, it will demonſtrate that there is but One *Northern* Pole; and that it is nearly where I place it: But if that Dip greatly *increaſe*, it will demonſtrate a ſecond Pole ſomewhere in thoſe Parts of *America*, where Dr. *Halley* places it. And to this Decretory Experiment do I appeal for a final Determination of this Queſtion. The Doctor ſeems to me to draw his Inferences from the *Variation*, which no Way proves any ſuch double Poles, as being full as ſenſible on our *Terrellæ*, which have no more than ſingle ones; while he avoids all Obſervations from the *Dip*, which are ſtill againſt him; and which are alone capable of diſcovering the exact Place of ſuch Poles, either upon the Surface of the Earth, or of *Terrellæ*. However, when one Set of Experiments with a Dipping-Needle, ſent to *Hudſon's Bay*, will certainly determine this Matter, 'tis a vain Thing to go on in the Way of Controverſy about it.

In ſhort, The Obſervations hitherto made, ſhew that the Foundations I go upon in this Diſcovery of the *Longitude* and the *Latitude* at Sea, are true and right: That the Terreſtrial Magnetiſm is very regular and uniform, in the open Seas; that the Latitude in the *Northern* Parts may even, without any Avoidance of the Shaking of the Ship, in ordinary calm Weather, be in good Degree thereby diſcovered already; and that if I can ſufficiently avoid the Shaking of the Ship, which I am now endeavouring, and have great

Hopes of performing, both *Latitude* and *Longitude* may by this Method be diſcovered in the greateſt Part of the ſailing World. I ſay nothing here of another Method of Trial, which I am alſo purſuing, and which depends, like this, on the avoiding the main Part of the Ship's Agitation; and if effected will be more eaſy and univerſal than this. But as to giving any farther Account of that to the Publick, unleſs it ſucceed, I have no Intention at all.

*N. B.* The original Journals are all in the Hands of my great Friend and Patron *Samuel Molyneux*, Eſq; Secretary to his Royal Highneſs, the Prince of *Wales*, and Fellow of the Royal Society: Which Journals, when I have compleated the reſt of the Obſervations I hope to procure, I intend to publiſh entire, for the more full Satisfaction of the curious.

*A Table of the Angle of Inclination below the Horizon, in* Dipping-Needles, *to every* $_{90}$ *Part of their reſpective equal Diſtances from the Magnetick Poles and Equator.*

| Diſt. from the Pole. | Dip. | | Diſt. from the Equat | Dip. | |
|---|---|---|---|---|---|
| ° | ° | ° | ° | ° | ° |
| 1 | 89 | 30 | 1 | 08 | 41 |
| 2 | 89 | 00 | 2 | 12 | 23 |
| 3 | 88 | 27 | 3 | 15 | 14 |
| 4 | 87 | 59 | 4 | 17 | 41 |
| 5 | 87 | 29 | 5 | 19 | 51 |
| 6 | 86 | 59 | 6 | 21 | 50 |
| 7 | 86 | 38 | 7 | 23 | 41 |
| 8 | 85 | 58 | 8 | 25 | 24 |
| 9 | 85 | 27 | 9 | 27 | 2 |
| 10 | 84 | 57 | 10 | 28 | 36 |
| 11 | 84 | 27 | 11 | 30 | 6 |
| 12 | 83 | 56 | 12 | 31 | 32 |
| 13 | 83 | 26 | 13 | 32 | 55 |

| Dift. from the Pole. | Dip. | | Dift. from the Pole. | Dip. | |
|---|---|---|---|---|---|
| ° | ° | ° | ° | ° | ° |
| 14 | 82 | 55 | 14 | 34 | 15 |
| 15 | 82 | 24 | 15 | 35 | 33 |
| 16 | 81 | 54 | 16 | 36 | 50 |
| 17 | 81 | 23 | 17 | 38 | 4 |
| 18 | 80 | 52 | 18 | 39 | 16 |
| 19 | 80 | 21 | 19 | 40 | 28 |
| 20 | 79 | 49 | 20 | 41 | 37 |
| 21 | 79 | 18 | 21 | 42 | 45 |
| 22 | 78 | 47 | 22 | 43 | 54 |
| 23 | 78 | 16 | 23 | 44 | 58 |
| 24 | 77 | 44 | 24 | 46 | 2 |
| 25 | 77 | 12 | 25 | 47 | 6 |
| 26 | 76 | 41 | 26 | 48 | 9 |
| 27 | 76 | 8 | 27 | 49 | 10 |
| 28 | 75 | 36 | 28 | 50 | 12 |
| 29 | 75 | 4 | 29 | 51 | 12 |
| 30 | 74 | 32 | 30 | 52 | 12 |
| 31 | 73 | 59 | 31 | 53 | 11 |
| 32 | 73 | 26 | 32 | 54 | 9 |
| 33 | 72 | 54 | 33 | 55 | 6 |
| 34 | 72 | 20 | 34 | 56 | 1 |
| 35 | 71 | 47 | 35 | 57 | 0 |
| 36 | 71 | 14 | 36 | 57 | 56 |
| 37 | 70 | 39 | 37 | 58 | 52 |
| 38 | 70 | 5 | 38 | 59 | 47 |
| 39 | 69 | 31 | 39 | 60 | 41 |
| 40 | 68 | 57 | 40 | 61 | 35 |
| 41 | 68 | 22 | 41 | 62 | 49 |
| 42 | 67 | 47 | 42 | 63 | 22 |
| 43 | 67 | 12 | 43 | 64 | 15 |
| 44 | 66 | 36 | 44 | 65 | 8 |
| 45 | 66 | 00 | 45 | 66 | 0 |

*N. B.* I take the *Northern* Pole of the Terreſtrial Magnet to be about the Meridian of *Archangel*, in the Latitude of $75\frac{1}{2}$. Its Equator to be nearly a great Circle, interſecting the Earth's Equator about $2\frac{1}{2}$ Degrees *Eaſtward* of the Meridian of *London*; and in its oppoſite Point. And that its utmoſt Latitude *Northward* is in the Gulph of *Bengall* about $12\frac{1}{2}$ Degrees; and as much *South* in the oppoſite Point, in the great *South Sea*. And that the *Southern* Pole is nearly circular; its Radius 40 Degrees of a great Circle, and its Center in a Meridian *Eaſtward* from *Ceilon* about $4\frac{1}{2}$ Degrees, and about $68\frac{1}{2}$ Latitude.

*N. B. London* is nearly $\frac{21}{54}$ = $\frac{35}{90}$ diſtance from the *North* Pole of the Magnet, whence its Dip will be at $74\frac{3}{4}$, which is certainly ſo in Fact. *Boſton* in *New-England* is [illegible] = [illegible], whence its Dip will be about 68° 22′, which Captain *Beal* found to be ſo in Fact. *Barbados* is about [illegible] = $\frac{22}{90}$ diſtant from the Equator of the Magnet, whence its Dip ought to be about 44° $\frac{1}{3}$, as Captain *Beal* alſo found it to be in Fact. *St. Helena* is about $\frac{14}{49}$ = [illegible], whence its Dip ought to be about 47° 50′ as Dr. *Halley* found it to be in Fact. And ſo every where in the main Ocean, at conſiderable Diſtances from the Shores.

*N. B.* If the Dip of any Needles be ſomewhat different at *London*, add or ſubſtract a proportionable Part of the Dip elſewhere. And you will have nearly the true Dip at any other Place with that Needle. Thus if your Needle differ from the other 2° or 120′, and ſhew the Dip at *London* 72° 45′ inſtead of 74° 45′, which is its proper Dip in this Table; and you require the true Dip by this Needle for

*Boſton* in *New-England*, *Southward*; which in the Table is 68° 22′, proceed thus. Becauſe the equal Diſtance of *Boſton* from the Magnetick Equator is 49 Parts of $60_{\llcorner}4$, the like Diſtance of *London* from that Equator; deduct $\frac{49}{60_{\llcorner}4}$ 120′ = 97′ = 1° 37′ out of the Tabular Dip 68° 22′. The Remainder is 66° 45′, for the true Dip at *Boſton* with that Needle. Thus if you want the true Dip, by the ſame Needle, at *Dronthem* in *Norway*, *Northward*: Becauſe the equal Diſtance of *Dronthem* from the Magnetick Pole is $15_{\llcorner}2$ Parts of $29_{\llcorner}6$ the Diſtance of *London* from that Pole; deduct $\frac{15_{\llcorner}2}{29_{\llcorner}6}$ 12′ = 62′ = 1° 2′ out of the Tabular Dip 82° 30′, and the Remainder, 81° 28′ is the true Dip at *Dronthem*, with that Needle: And ſo in all other Caſes whatſoever.

*N. B.* The Table before ſet down, ſuppoſes that the true Dip differs according to ſuch a Line of Sines, whoſe middle Point gives 66° on one Side, and 24 on the other; and is made by adding or ſubſtracting 8 to the Complement of the Dip found by the natural Sines for every $\frac{1}{5}$ of equal Diſtances from the Equator or Pole.

*N. B.* If any deſire to calculate by Trigonometry the Diſtances of all Places from the magnetick Equator of Poles, and the Diſtances of that Equator and thoſe Poles in every particular Caſe, both made uſe of in the foregoing Calculations, it is thus to be done:

In the (*Fig.* 12.) Triangle $BLA$ we have $BL$ the Co-Latitude of *London*; $BA$ the Co-Latitude of the magnetick *North* Pole; and the included Angle, $ABL$ = the Diſtance of the Meridian of that Pole from the Meridian of *London*; to find the Angle $\mathcal{Q}AM$ and the Side $AL$. Then in the Triangle *QAM*, we have the Angles *QAM* and *QMA*, and the Side *AM*, = the Diſtance of the Magnetick Pole from the Magnetick Equator, to find *AQ*. So we have the Proportion of *AL* to *AQ*, *Q.E.I.*

But ſince the *Data* are not yet ſufficiently exact for the Calculation, meaſuring is ſufficient.

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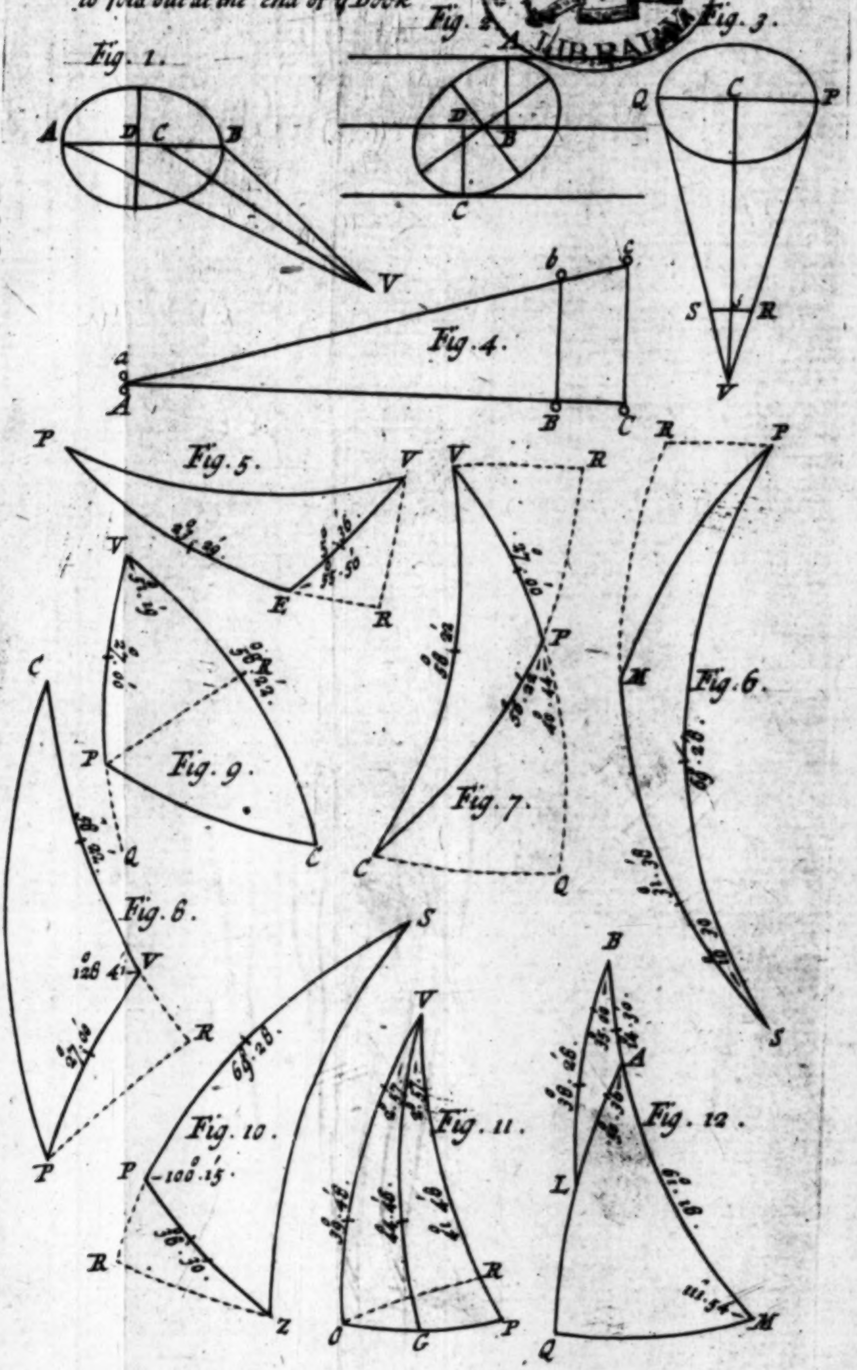

# ERRATA.

P*Age* 1. *lines* 12, 13. *read*, 5 Leap Days; and with 11 Days when 4 Leap Days. *P*. 7. *over againſt* 21, &c. *read*, 53. 2. 55. 40. 58. 19. 60. 58. 63. 38. 66. 19. 69. 0. 71. 42. 74. 24. 77. 7. 79. 50. 82. 34. 85. 19. 88. 4. 90. 50. *P*. 10. *l*. 32. *read*, Summer. *P*. 13. *l*. 7. *r*. $10^d$. *l*. 8. *and* 10, *and* 16. *r*. 18. *P*. 13. *l*. 24. *r*. *Bern*, *Zurich*, and *Pillaw* near *Koningsberg* in *Pruſſia*. *Dele p*. 16. *l*. 32. *to p*. 17. *l*. 10. *and inſtead of it read thus:* Its greateſt Alteration therefore muſt be at the mean Diſtance; and is the Difference of the Equation belonging there to the Addition of $10^\circ.8' = 17$. which Space the Moon uſually goes in about 36 of time. So that the Difference on this Account muſt, each Period, be uſually leſs than $36'$. And as to the Moon's own Motion, it has alſo its greateſt Alteration at its mean Diſtance; and is the Difference of the Equation at $2^\circ. 51 . \frac{1}{2} = 17$. which the Moon uſually goes in about $36'$. of time. So that the Difference on this Account muſt, each Period, be uſually leſs than $36'$. alſo, and on both Accounts leſs than $1^h. 12'$.

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